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## Number and Quantity

### The Real Number System

A1: N-RN

#### B. Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

### Quantities★

A1: N-Q

#### A. Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

## Algebra

### Seeing Structure in Expressions

A1: A-SSE

#### A. Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context. ★
  - a. Interpret parts of an expression, such as terms, factors, and coefficients.
  - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret  $P(1+r)^n$  as the product of  $P$  and a factor not depending on  $P$ .*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ , or see  $2x^2 + 8x$  as  $(2x)(x) + 2x(4)$ , thus recognizing it as a polynomial whose terms are products of monomials and the polynomial can be factored as  $2x(x+4)$ .*

#### B. Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
  - a. Factor a quadratic expression to reveal the zeros of the function it defines.
  - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
  - c. Use the properties of exponents to transform expressions for exponential functions emphasizing integer exponents. *For example, the growth of bacteria can be modeled by either  $f(t) = 3^{(t+2)}$  or  $g(t) = 9(3^t)$  because the expression  $3^{(t+2)}$  can be rewritten as  $(3^t)(3^2) = 9(3^t)$ .*

### Arithmetic with Polynomials and Rational Expressions

A1: A-APR

#### A. Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

#### B. Understand the relationship between zeros and factors of polynomials.

- Identify zeros of quadratic functions, and use the zeros to sketch a graph of the function defined by the polynomial.

### Creating Equations\*

A1: A-CED

#### A. Create equations that describe numbers or relationships.

- Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear, quadratic, and exponential functions.*
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
- Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .*

### Reasoning with Equations and Inequalities

A1: A-REI

#### A. Understand solving equations as a process of reasoning and explain the reasoning.

- Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

#### B. Solve equations and inequalities in one variable.

- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- Solve quadratic equations in one variable.
  - Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.
  - Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as "no real solution."

#### C. Solve systems of equations.

- Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

#### D. Represent and solve equations and inequalities graphically.

- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, piecewise linear (to include absolute value), and exponential functions.\*
- Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

## Functions

### Interpreting Functions

A1: F-IF

#### A. Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions whose domain is a subset of the integers. Relate arithmetic sequences to linear functions and geometric sequences to exponential functions.

#### B. Interpret functions that arise in applications in terms of the context.

4. For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.* ★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.* ★
6. Calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

#### C. Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
  - a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
  - b. Graph piecewise linear (to include absolute value) and exponential functions.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
9. Compare properties of two functions (linear, quadratic, piecewise linear [to include absolute value] or exponential) each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, determine which has the larger maximum.*

**Building Functions**

**A1: F-BF**

**A. Build a function that models a relationship between two quantities.**

1. Write a linear, quadratic, or exponential function that describes a relationship between two quantities. ★
  - a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

**B. Build new functions from existing functions.**

3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative). Without technology, find the value of  $k$  given the graphs of linear and quadratic functions. With technology, experiment with cases and illustrate an explanation of the effects on the graph that include cases where  $f(x)$  is a linear, quadratic, piecewise linear (to include absolute value), or exponential function.

**Linear, Quadratic, and Exponential Models★**

**A1: F-LE**

**A. Construct and compare linear, quadratic, and exponential models and solve problems.**

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
  - a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
  - c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

**B. Interpret expressions for functions in terms of the situation they model.**

5. Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.

**Statistics and Probability★**

**Interpreting Categorical and Quantitative Data**

**A1: S-ID**

**A. Summarize, represent, and interpret data on a single count or measurement variable.**

2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

**B. Summarize, represent, and interpret data on two categorical and quantitative variables.**

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
  - a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and quadratic models.*
  - b. Informally assess the fit of a function by plotting and analyzing residuals.

c. Fit a linear function for a scatter plot that suggests a linear association.

**C. Interpret linear models.**

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

★Modeling Standards - Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards. The basic modeling cycle involves

1. identifying variables in the situation and selecting those that represent essential features,
  2. formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables,
  3. analyzing and performing operations on these relationships to draw conclusions,
  4. interpreting the results of the mathematics in terms of the original situation,
  5. validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable,
  6. reporting on the conclusions and the reasoning behind them.
- Choices, assumptions, and approximations are present throughout this cycle.