NEW MILFORD PUBLIC SCHOOLS

New Milford, Connecticut

# Algebra 3

February/2023

BOE Approved June 2023

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## New Milford's Mission Statement

The mission of the New Milford Public Schools, a collaborative partnership of students, educators, family and community, is to prepare each and every student to compete and excel in an ever-changing world, embrace challenges with vigor, respect and appreciate the worth of every human being, and contribute to society by providing effective instruction and dynamic curriculum, offering a wide range of valuable experiences, and inspiring students to pursue their dreams and aspirations.

BOE Approved June 2023

# Algebra 3

## Grade 11 & 12

After a review of the more challenging topics of second year algebra (factoring, quadratic equations, rational expression simplification, logarithms, etc.), this course will cover topics in rational functions, trigonometry, and the unit circle in preparation for college placement tests. A graphing calculator will be used in this course and is used extensively throughout the year.

## Vision of the Graduate

Algebra 3 is a course that promotes critical thinking, problem solving, and good communication. Throughout this course students will need to analyze and find the best process to solve problems. They will be expected to communicate technically and verbally to explain how they arrived at their solution and what the solution means in the context of an application. Students will persevere through solving problems and will collaborate with their peers.

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## Pacing Guide

The following units are suggested guidelines for activities and materials. The intention of this course is to review concepts in advanced algebra and to introduce trigonometry as a way to strengthen student understanding of the topics and to prepare them for college placement tests as well as for college mathematics courses. Flexibility and creativity should be used throughout the course.

( Based on a block schedule)

| Unit # | Title | Days |
|---|---|---|
| 1 | Fundamental Concepts of Algebra, Part 1 | 15 days |
| 2 | Fundamental Concepts of Algebra, Part 2 | 15 days |
| 3 | Graphs, Functions, and Models | 11 days |
| 4 | Polynomial and Rational Functions | 18 days |
| 5 | Exponential and Logarithmic Functions | 8 days |
| 6 | Trigonometric Functions | 15 days** |
| 7 | Law of Sines/Cosines | 7 days ** |

**As time allows

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Unit 1: Fundamental Concepts of Algebra, Part 1 (Preliminary Chapter sections 2-5)

**CC.9-12.N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
**CC.9-12.A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
**CC.9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**CCSS.Math.Practice.MP2** Reason abstractly and quantitatively.
**CCSS.Math.Practice.MP5** Use appropriate tools strategically.
**CCSS.Math.Practice.MP6** Attend to precision

***Transfer***

*Students will be able to independently use their learning to…*

- Analyze a problem and apply the appropriate techniques to simplifying expressions
- Reason abstractly
- Justify their reasoning or understanding by explaining
- Attend to precision when making mathematical statements

***Meaning***

UNDERSTANDINGS

*Students will understand that…*

Values, expressions, and polynomials can be simplified using a specific process.

Exponents and radicals are related to the operation of multiplication.

Polynomials can be added, subtracted, and multiplied to make a more simplified expression.

Polynomials can be broken up into products of more simplified terms by factoring.

ESSENTIAL QUESTIONS

*Students will keep considering…*

How are the properties of exponents related to the basic arithmetic operations?

How do radicals relate to exponents?

How are polynomial expressions combined using operations of addition, subtraction, and multiplication?

Why do some expressions such as radical and rational expressions have restrictions under the set of real numbers?

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| | ***Acquisition*** | |
| | *Students will know…*<br><br>● There are specific properties of exponents to help perform operations among terms with exponents<br>● A radical is the inverse of an exponent and the specific process to simplifying radical expressions<br>● Polynomials can be classified according to their degree, and that degree tells us information about the polynomial such as the specific number of factors or solutions associated with that polynomial<br>● Polynomials can be combined under the operations of addition, subtraction, and multiplication<br>● Factoring polynomials is a method of breaking a polynomial in the the product of smaller, first degree polynomials | *Students will be skilled at…*<br><br>● Simplifying expressions using the properties of exponents<br>● Simplifying radical expressions<br>● Adding, subtracting, multiplying and dividing radical expressions<br>● Identifying the degree of a polynomial<br>● Adding, subtracting, and multiplying polynomials<br>● Applying the steps to completely factor polynomial expressions |

| Code | Evaluative Criteria | Assessment Evidence |
|---|---|---|
| T, M, A | Evaluative Criteria consists of<br>● an explanation of how information was evaluated and analyzed<br>● correct calculations of information<br>● accurate, clear display of the analysis<br>● a conclusion that provides the required information to the insurance director | PERFORMANCE TASK(S):<br><br>Goal/challenge -Apply polynomial expressions to real world situations<br>Role for student: Analyst<br><br>Audience for student work: Director of an insurance company<br><br>Situation - Student will be given real world situations related to populations that must be analyzed in order to make predictions<br><br>Products and performances generated by student - Student made presentation (poster, google slide, etc) with specific predictions on given populations, with work to support these predictions<br><br>Standards/criteria for judging success:<br>-The mathematical processes are shown for analysis of information<br>-Accurate calculations<br>- A neat, organized, clear presentation of the information |

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| T, M, A<br><br>M, A<br><br>M, A | Evaluative criteria consists of:<br><br>● Is the correct method used to solve the problem<br><br>● Is the analysis of the problem accurate<br><br>● Is the mathematics completed correctly | OTHER EVIDENCE:<br>*Students will show they have achieved Stage 1 goals by…*<br><br>● Alternative assessment projects such as posters, drawings, pictures and real world applications<br><br>● Review of standardized test questions to prep students for the challenge of the SAT and college placement tests<br><br>● Performance tasks modeling real world and application problems<br><br>● Quizzes<br><br>● Unit Test - to include a variety of DOK level problems and may include SAT style problems. |

| Code | *Pre-Assessment* | |
|---|---|---|
| T, M<br><br>T, M, A | ● Teacher checks for prerequisite skills and prior knowledge via warm-up and questioning activities, such as basic problems on order of operations, solving one and two step equations and properties of exponents<br>● Prerequisite knowledge will be reviewed as it is incorporated into topics both in class and on review assignments | |
| M, T, A<br><br>T, M, A<br><br>M, A<br><br>T, M, A<br><br>M, A<br><br>T, M, A | ● Teacher will check for prerequisite and prior knowledge via warm-up and questioning activities, such as basic review problems on exponents, radicals, polynomial operations, and factoring. As the lessons progress, students can also be given questions such as "find the mistakes in simplifying a rational expression," "explain how to solve a problem " Additionally, warm ups should contain review on previous material covered during the unit to ensure retention and mastery.<br>● Teacher will introduce the properties of exponents as "short cuts" to what exponents mean (ex: $x^3 \cdot x^2 = x^5$ since $x^3$ = xxx, $x^2$ = xx, giving a "string" of 5 x's). As properties are introduced, more complex problems will be given to simplify as a class.<br>● Students will work independently and as a class simplifying expressions with exponents.<br>● Teacher will review radicals and model how to simplify them. As lesson progresses from simplifying single radicals to multiplying, adding, and subtracting, to rationalizing, to fractional exponents, teacher will lead class through each process, having students share ideas and thoughts on the process.<br>● Students will simplify radicals by working in teacher created groups and/or independently.<br>● Teacher will model factoring of a polynomial expression, beginning with GCF and grouping one day, then difference of squares and sum and difference of cubes the next day, then trinomials. Teacher will allow students many | Progress Monitoring<br><br>● Warm up questions<br><br>● Monitoring class work through board work, group work, questioning, and walk-arounds<br><br>● Check for understanding via going over homework and medium such as reflections and exit tickets<br><br>● Class worksheets with direct teacher observation or self assessment<br><br>● Practice on whiteboard/chalkboard with direct teacher observation<br><br>● Kahoot quizzes with review questions and direct teacher observation<br><br>● Google form or other review assignments<br><br>● Homework assignments with direct teacher observation or self assessment<br><br>● Differentiate through purposeful or flexible grouping, use of diagrams and explanations to demonstrate understanding and active |

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| M, A<br><br>M, A | opportunities for independent practice and teacher created groups throughout this topic.<br>● Students will use the white boards to practice factoring polynomial expressions.<br>● Students will create a poster that explains the steps to factoring, and includes an example with each step.<br><br>All Resources and materials must adhere to all New Milford Board of Education policies and regulations and are subject to New Milford Board of Education approval. Resources and materials must be researched and vetted by the writers and department heads prior to submission for approval.<br><br>● Textbook: Blitzer, Robert. Precalculus Second Edition, Upper Saddle River, NJ: Pearson, 2004.<br>● Supplemental activities from the textbook resources<br>● Teacher-made supplemental activities on applications, performance tasks, and chapter review<br>● Graphing calculator TI Emulator software.<br>● On-line resources such as YouTube, Khan Academy, Desmos, EdPuzzle, Kahoot, etc. | lessons involving discovery, scaffolding, jigsaw activities and use of hands-on manipulatives<br><br>● Summative assessments<br>Quizzes<br>Unit test |

**CC.9-12.N.CN.7** Solve quadratic equations with real coefficients that have complex solutions.
**CC.9-12.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression
**CC.9-12.A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
**CC.9-12.A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
**CC.9-12.A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
**CC.9-12.A.REI.4** Solve quadratic equations in one variable.
**CC.9-12.A.REI.4b** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as

***Transfer***

*Students will be able to independently use their learning to…*

- Analyze a problem and apply the appropriate techniques to simplifying expressions
- Reason abstractly
- Justify their reasoning or understanding by explaining
- Attend to precision when making mathematical statements

***Meaning***

UNDERSTANDINGS

*Students will understand that…*

- Rational expressions can be simplified, added, subtracted, multiplied, and divided using the same concepts as with operations on fractions.
- Problems can have values that must be excluded from being possible solutions.
- Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole.
- Literal equations can represent many real world situations.
- Absolute value represents a positive distance from zero on a number line and may result in two solutions.
- Quadratic equations can be solved using a variety of methods, specifically

ESSENTIAL QUESTIONS

*Students will keep considering…*

- Why do some expressions such as radical and rational expressions have restrictions under the set of real numbers?
- How can rational expressions be simplified?
- How are algebraic operations and notation used to simplify and solve equations and inequalities?
- How do literal equations apply to real world situations?
- How can quadratic equations be solved?
- How do absolute value inequalities relate to real world situations?

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| appropriate to the initial form of the equation<br><br>**CCSS.Math.Practice.MP2** Reason abstractly and quantitatively.<br>**CCSS.Math.Practice.MP5** Use appropriate tools strategically.<br>**CCSS.Math.Practice.MP6** Attend to precision | factoring, the quadratic formula, completing the square, and the square root method. | |
| | ***Acquisition*** | |
| | *Students will know…*<br><br>● The domain of a rational expressions sometimes has restrictions due to constraints on the problems (such as: the denominator of a fraction cannot be zero)<br>● Rational Expressions require a specific processes to be utilized when simplifying and performing operations<br>● Linear, rational, literal, and absolute value equations have specific characteristics and unique methods to being solved<br>● A quadratic equation is of degree two and can be solved by the methods of factoring, completing the square, the quadratic formula, and the square root method<br>● Absolute value inequalities can be related to the concept of distance on a number line | *Students will be skilled at…*<br><br>● Identifying values that are restricted from the domain of a rational expression.<br>● Simplifying, adding, subtracting, multiplying, and dividing rational expressions.<br>● Simplifying complex fractions<br>● Identifying and knowing how to solve linear, rational, and literal equations<br>● Solving quadratic equations using the square root method, factoring, quadratic formula, and by completing the square<br>● Solving absolute value inequalities<br>● Solving real world and verbal problems using methods listed above |

| Code | Evaluative Criteria | Assessment Evidence |
|---|---|---|
| T, M, A | Evaluative Criteria consists of<br>● correct analyses of whether each step is correct or incorrect<br>● a clear and accurate explanation of why each step is correct or incorrect<br>● feedback contains appropriate mathematical vocabulary | PERFORMANCE TASK(S):<br>Goal: To identify correct and incorrect steps to solving an algebraic equation<br>Role: Teacher<br>Audience: Student who solved the problem<br>Situation: Students are given a problem with specific steps shown as a solution. Students then identify if each step is correct or incorrect, and explain why.<br>Product: Corrected problem, to include feedback and explanation. |

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| M, A<br><br>T, M<br><br>T, M, A<br><br>M, A | Evaluative criteria consists of:<br><br>● Is the correct method used to solve the problem<br><br>● Is the analysis of the problem accurate<br><br>● Is the mathematics completed correctly<br><br>● Are students using correct mathematical terminology | OTHER EVIDENCE:<br><br>● Alternative assessment projects such as posters, drawings, pictures and real world applications<br><br>● Review of standardized test questions to prep students for the challenge of the SAT and college placement tests<br><br>● Performance tasks modeling real world and application problems<br><br>● Quizzes<br><br>● Unit Test - to include a variety of DOK level problems and may include SAT style problems. |

| Code | ***Pre-Assessment*** | |
|---|---|---|
| **T, M** | ● Teacher checks for prerequisite skills and prior knowledge via warm-up and questioning activities, such as performing operations with basic fractions, solving linear equations (ex: putting an equation in the y=mx+b form),<br>● Prerequisite knowledge will be reviewed as it is incorporated into topics both in class and on review assignments | |
| **T, M** | | |
| | Summary of Key Learning Events and Instruction | Progress Monitoring |
| T, M, A | ● Connections will be made to the similarities in the process used with rational expressions versus basic fractions. Teacher will explain the specific steps to the process of simplifying rational expressions. Teacher will allow much time for practice with each operation and will review the differences in the steps required for each type of problem. Supplemental worksheets and board problems should be used to assess mastery of this process. | ● Warm up questions<br>● Monitoring class work through board work, group work, questioning, and walk-arounds<br>● Check for understanding via going over homework and medium such as reflections and exit tickets |
| M, A | ● Students will verbally explain the process to add and subtract rational expressions. | ● Class worksheets with direct teacher observation or self assessment |
| T, A | ● Students will work independently and in small groups to practice simplifying rational expressions. | |
| T, M, A | ● Teacher will review the concept of absolute value as it relates to the number line. Distance interpretation will be used as an option to solving absolute value equations and inequalities. | ● Practice on whiteboard/chalkboard with direct teacher observation |
| T, A | ● Students will practice solving linear equations, literal equation, and absolute value equations and inequalities as a class and independently. | ● Kahoot quizzes with review questions and direct teacher observation<br>● Google form or other review assignments |
| T, M, A | ● Teacher will discuss the need for alternative methods for solving quadratic equations. Modeling of examples that do not factor should be used to show the methods of completing the square and the quadratic formula. Ample time and practice of each method will be given. | ● Homework assignments with direct teacher observation or self assessment<br>● Differentiate through purposeful or flexible grouping, use of diagrams and explanations |

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| M, A<br><br>M, A<br><br>A | ● Students will solve quadratic equations using the methods of factoring, quadratic formula, completing the square, and the square root method by working in pairs.<br>● Teacher will use song and/or a “story” as a way to help students memorize the quadratic formula.<br>● Students will verbally state the quadratic formula from memory, and may use song or a story as a way of stating it.<br><br>All Resources and materials must adhere to all New Milford Board of Education policies and regulations and are subject to New Milford Board of Education approval. Resources and materials must be researched and vetted by the writers and department heads prior to submission for approval.<br><br>● Textbook: Blitzer, Robert. Precalculus Second Edition, Upper Saddle River, NJ: Pearson, 2004.<br>● Supplemental activities from the textbook resources<br>● Teacher-made supplemental activities on applications, performance tasks, and chapter review<br>● Graphing calculator TI Emulator software.<br>● On-line resources such as YouTube, Khan Academy, Desmos, EdPuzzle, Kahoot, etc. | to demonstrate understanding and active lessons involving discovery, scaffolding, jigsaw activities and use of hands-on manipulatives<br><br>● Summative assessments<br>Quizzes<br>Unit test |

Unit 3: Graphs, Functions, and Models (Chapter 1)

<table>
<tr><td>ESTABLISHED GOALS<br><br>● CC.9-12.F.1F.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*<br><br>● CCSS.Math.Practice.MP 2 Reason abstractly and quantitatively.<br>● CCSS.Math.Practice.MP 5 Use appropriate tools strategically.<br>● CCSS.Math.Practice.MP 6 Attend to precision</td><td colspan="2">Transfer<br><br>Students will be able to independently use their learning to…<br>● Analyze a problem and apply the appropriate techniques to simplifying expressions<br>● Reason abstractly<br>● Justify their reasoning or understanding by explaining<br>● Attend to precision when making mathematical statements<br><br>Meaning</td></tr>
<tr><td></td><td>UNDERSTANDINGS<br>Students will understand that…<br>● Linear, quadratic, absolute value, and cubic functions have different shapes when graphed.<br>● Real world situations can often be represented graphically.<br>● Intercepts have special meanings in graphs.<br>● Linear equations can be represented in many forms.<br>● The slope of a line represents the rate of change for a specific relation.<br>● The distance formula and equation of a circle are derived from the Pythagorean Theorem.<br>● Functions are special relations and have a domain and a range.</td><td>ESSENTIAL QUESTIONS<br>Students will keep considering…<br>● How does the relationship between variables help us to understand real world situations?<br>● How does visualizing values as points on a graph help us?<br>● What are the relationships between equations of functions and their graphs?<br>● What types of real world problems can be modeled with a linear equation?<br>● Why is slope useful?<br>● Why is it important to understand a circle as both a geometric shape and an algebraic equation?<br>● Why do some functions have restricted values?</td></tr>
</table>

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| | ● Functions can have restrictions on their domain.<br>● Graphs of functions can be used to find the domain, range, intercepts, and to tell the nature of the function(increasing, decreasing, constant). | |
| | ***Acquisition*** | |
| | *Students will know…*<br><br>● Slope represents the rate of change and describes the slant of a line<br>● The different equations of lines and how to write the equation of a line given specific information.<br>● The distance and midpoint formulas.<br>● The equation of a circle and how to graph a circle.<br>● A function is a specific type of equation where every input value is paired with exactly one output value<br>● Functions have a domain, range, and function notation can be used to write functions<br>● What the graph of linear, absolute value, quadratic, cubic, and square root functions look like.<br>● Composition of functions combines two or more functions and follows a specific process<br>● The inverse of a function “undoes” what a function “did” to a value to give the original value back.<br>● Key Terms: Intercept, line, slope, slope-intercept form, point-slope form, distance, midpoint, circle, center, radius, function, domain, range, | *Students will be skilled at…*<br><br>● Graphing various equations using different methods including making a T-table and by using the graphing calculator.<br>● Finding the slope of a line given two points or an equation.<br>● Graphing a line using slope-intercept form.<br>● Writing an equation of a line in slope-intercept, point-slope, and undefined slope form.<br>● Finding the distance and midpoint of a segment given the endpoints.<br>● Putting the equation of a circle in standard form in order to identify the center and radius, graph a circle from the equation, and write an equation given the graph.<br>● Identifying whether a graph or equation is a function, and to find the domain and range of a function.<br>● Identifying intercepts, points of relative maxima and minima, intervals where the function is increasing, decreasing, or constant, as well as find specific values from the graph of a function.<br>● Performing operations involving the composition of functions. |

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| | relation, function notation, relative maxima and minima, increasing function, decreasing function, constant function, inverse function, vertical line test, horizontal line test. | • Finding the inverse of a function. |

STAGE 2

| Code | Evaluative Criteria | Assessment Evidence |
|---|---|---|
| T, M, A | Evaluative Criteria consists of<br>● a presentation that is clear, organized, and thorough<br>● a clear and accurate explanation of their analysis of the equation, and how they found their predicted values<br>● work is shown and is correct | PERFORMANCE TASK(S):<br>Goal: To interpret real-world graphs, write equations from the graphs, and make predictions from that equation.<br>Role: Statistician<br>Audience: Managers of two companies<br>Situation: Students are given two graphs – one dealing with average cost of a retail prescription and the other dealing with the number of workers per Social Security beneficiary. Students are to identify and interpret the y-intercept, slope, linear equation, and explain the meanings of these values.<br>Product: Students will make a presentation (poster, slideshow) on their future predictions based on their findings |

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| A, M<br><br>A, M<br><br>A, M<br><br>T, A, M | Evaluative criteria consists of:<br><br>● Is the correct method used to solve the problem<br><br>● Is the analysis of the problem accurate<br><br>● Is the mathematics completed correctly<br><br>● Are students using correct mathematical terminology | OTHER EVIDENCE:<br><br>● Alternative assessment projects such as posters, drawings, pictures and real world applications<br><br>● Review of standardized test questions to prep students for the challenge of the SAT and college placement tests<br><br>● Performance tasks modeling real world and application problems<br><br>● Quizzes<br><br>● Unit Test - to include a variety of DOK level problems and may include SAT style problems. |

| Code<br><br>T, M<br><br>T, M, A | ***Pre-Assessment***<br>● Teacher checks for prerequisite skills and prior knowledge via warm-up and questioning activities, such as graphing points and linear equations, identifying the intercepts of graphs, and function basics.<br>● Prerequisite knowledge will be reviewed as it is incorporated into topics both in class and on review assignments | |
|---|---|---|
| | Summary of Key Learning Events and Instruction | Progress Monitoring |
| T, M | ● Teacher will have a quick review of graphing on the rectangular coordinate system, labeling points. Teacher will model graphing equations using a T-table (graphic organizer), and allow students class time to practice graphing various equations. | ● Warm up questions<br>● Monitoring class work through board work, group work, questioning, and walk-arounds |
| T, M, A | ● Students will work independently graphing different equations by making a T-table. | ● Check for understanding via going over homework and medium such as reflections and exit tickets |
| T, M | ● Teacher will use the graphing calculator or desmos to demonstrate how to graph equations. Teacher will discuss how to change the viewing window, and identifying critical values such as intercepts, maximum and minimum values. Modeling of application problems and interpreting data will be facilitated by the teacher. | ● Class worksheets with direct teacher observation or self assessment<br>● Practice on whiteboard/chalkboard with direct teacher observation |
| M, A | ● Students will practice using the graphing calculator or desmos to graph equations, change the viewing window, and find intervals and other values using various tools such as trace and table. Students will work in pairs graphing equations as well as reading and interpreting real-world graphs. | ● Kahoot quizzes with review questions and direct teacher observation<br>● Google form or other review assignments |
| T, M | ● Teacher will give a warm-up question about slope of a line. The concept of slope and different types of slope will be discussed. Slope formula will be written on the board and will be modeled. Teacher will also review the slope-intercept equation of a line and will then give students practice finding slope and graphing lines using the slope-intercept form. | ● Homework assignments with direct teacher observation or self assessment<br>● Differentiate through purposeful or flexible grouping, use of diagrams and explanations to demonstrate understanding and active lessons involving discovery, scaffolding, |

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| M, A | ● Students will calculate the slope of a line and graph lines using slope intercept form by working in teacher created small groups or independently. | jigsaw activities and use of hands-on manipulatives<br><br>● Summative assessments<br>Quizzes<br>Unit test |
| T, M, A | ● Teacher will review all equations of lines, and will model examples on how to write equations given various information. Class work will be given for independent practice. | |
| M, A | ● Students will write equations of lines given different situations. | |
| T, M, A | ● Teacher will review the Pythagorean Theorem as a way to lead into the derivation of the distance formula. Teacher will also review the midpoint formula. Both formulas will be written on the board, and problems will be modeled by the teacher. | |
| M, A | ● Students will work in teacher created groups to find the distance between two points and the midpoint of a segment. | |
| T, M, A | ● Teacher will relate the equation of a circle to the Pythagorean Theorem. The standard form will be given and teacher will model how to complete the square and how to find the center and radius and graph a circle. | |
| T, M, A | ● Students will find the center and radius of a circle, and to complete the square to get the equation in graphing form. Students will put answers on the board for other students to compare and discuss. | |
| T, M, A | ● Teacher will give a warm-up on functions to assess what students remember. Relation, function, domain, range, and function notation will be defined and identified from examples. Teacher will lead class through problems on evaluating functions for specific values. | |
| M, A | ● Students will identify functions and find their domain and range. Students will also evaluate functions. | |
| T, M, A | ● Teacher will review how to graph functions and then will facilitate a discussion on how to find the domain, range, intercepts, and values from the graph. Teacher will model | |

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| | how to identify increasing, decreasing and constant values using interval notation. | |
| M, A | ● Students will graph functions and find the domain, range, and other critical values from graphs of functions. Students will complete a review worksheet on functions and will put their answers on the board. | |
| T, M, A | ● Teacher will discuss the origins of the infinity symbol as it relates to the mobius strip. Teacher will then lead class through hands-on activities pertaining to the mobius strip and its properties. | |
| M, A | ● Students will view a video an then complete an activity on the mobius strip. | |
| T, M, A | ● Teacher will model the process to find the composition of functions and inverse of a function. Teacher will give problems for students to complete at their seats or at the board. | |
| M, A | ● Students will work independently and in pairs to find the composition of functions and the inverse of functions. | |
| | All Resources and materials must adhere to all New Milford Board of Education policies and regulations and are subject to New Milford Board of Education approval. Resources and materials must be researched and vetted by the writers and department heads prior to submission for approval. | |
| | ● Textbook: Blitzer, Robert. Precalculus Second Edition, Upper Saddle River, NJ: Pearson, 2004.<br>● Supplemental activities from the textbook resources<br>● Teacher-made supplemental activities on applications, performance tasks, and chapter review<br>● Graphing calculator TI Emulator software.<br>● On-line resources such as YouTube, Khan Academy, | |

| | | |
|---|---|---|
| | Desmos, EdPuzzle, Kahoot, etc. | |

Unit 4: Polynomial and Rational Functions (Chapter 2)

ESTABLISHED GOALS

- **CC.9-12.N.CN.1** Know there is a complex number $i$ such that $i^2 = \sqrt{-1}$, and every complex number has the form $a + bi$ with $a$ and $b$ real
- **CC.9-12.N.CN.3 (+)** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers
- **CC.9-12.A.APR.2** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
- **CC.9-12.A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **CC.9-12.F.IF.7**: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

***Transfer***

*Students will be able to independently use their learning to…*

- Analyze a problem and apply the appropriate techniques to simplifying expressions
- Reason abstractly
- Justify their reasoning or understanding by explaining
- Attend to precision when making mathematical statements

***Meaning***

UNDERSTANDINGS

*Students will understand that…*

- The value $\sqrt{-1}$ can be represented as an imaginary number (i).
- Complex numbers combine real and imaginary numbers and can have operations of addition, subtraction, multiplication, and division performed on them.
- Quadratic functions are shaped like parabolas and have special properties.
- Polynomial functions can be graphed to find zeros, maximum and minimum values
- Long and synthetic division can be used to find the zeros of a polynomial.
- The remainder and factor theorems help find specific values of a

ESSENTIAL QUESTIONS

*Students will keep considering…*

- Why are some values not considered real numbers?
- Where did complex numbers originate and how do they fit into the algebraic framework?
- How do quadratic functions relate to real world situations?
- How can technology be used to represent functions and to verify solutions found manually?
- What is the importance of finding values such as intercepts and maximum/minimum from a graph?
- Why do different functions have different graphs and behaviors?

| | | |
|---|---|---|
| ● **CCSS.Math.Practice.MP 2** Reason abstractly and quantitatively.<br>● **CCSS.Math.Practice.MP 5** Use appropriate tools strategically.<br>● **CCSS.Math.Practice.MP 6** Attend to precision<br>● | polynomial, and are used to check if a value is a zero.<br>● Rational functions have special behaviors when graphed, and have undefined values in their domain and range, leading to asymptotic behavior | |
| | ***Acquisition*** | |
| | *Students will know…*<br>● The definition of imaginary and complex numbers and how to perform operations with complex numbers<br>● How to define, write the equation in graphing form, and graph quadratic functions<br>● Polynomial functions have special characteristics and can be graphed by finding their zeros and end behaviors<br>● Long and synthetic division can be used to find the zeros of a polynomial<br>● The Remainder and Factor Theorems can be used to find the zeros of a polynomial when it cannot be factored<br>● The general curves of rational functions, how to find the asymptotes, intercepts, and sketch a graph<br>● Key Terms: imaginary number, complex number, quadratic function, parabola, vertex, axis of symmetry, x- and y- intercepts, domain, range, polynomial function, zeros, roots, long division, synthetic division, Remainder Theorem, Factor Theorem, rational function, asymptote. | *Students will be skilled at…*<br>● Simplifying imaginary and complex numbers and performing mathematical operations with them.<br>● Graphing quadratic functions, and identifying the vertex, axis of symmetry, direction of opening, maximum or minimum value, x- and y- intercepts, domain and range.<br>● Completing the square to get a quadratic function in graphing form.<br>● Graphing polynomial functions by finding the zeros and identifying the nature of the curve.<br>● Using long and synthetic division to divide polynomials and find the remainder.<br>● Applying the Remainder and Factor Theorems to find the zeros of a polynomial, or to tell if a value is a zero.<br>● Graphing a rational function by finding the vertical, horizontal, and/or slant asymptotes, and the x- and y- intercepts. |

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STAGE 2

| Code | Evaluative Criteria | Assessment Evidence |
|---|---|---|
| T, M, A<br><br>T, M, A<br><br>T, M, A | Evaluative Criteria consists of<br><br>● critical values (asymptotes, intercepts) are calculated correctly<br>● appropriate work is shown clearly<br>● product is a neat, correct graph with critical values shown and labeled | PERFORMANCE TASK(S):<br>*Students will show that they really understand evidence of…*<br><br>Goal: To produce a graph of a unique rational function<br>Role: Engineer<br>Audience: Head of Engineering Department<br>Situation: Students are given a unique rational function. They are to identify all key features of the rational function, and produce an accurate graph that models that function<br>Product: Students will bring the graph “to life” by using graph paper, construction paper, wiki sticks, linguini and other materials |

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|---|---|---|
| A, M<br><br>A, M<br><br>A, M<br><br>T, A, M | Evaluative Criteria consists of:<br><br>● Is the correct method used to solve the problem<br><br>● Is the analysis of the problem accurate<br><br>● Is the mathematics completed correctly<br><br>● Are students using correct mathematical terminology | OTHER EVIDENCE:<br><br>● Alternative assessment projects such as posters, drawings, pictures and real world applications<br><br>● Review of standardized test questions to prep students for the challenge of the SAT and college placement tests<br><br>● Performance tasks modeling real world and application problems<br><br>● Quizzes<br><br>● Unit Test - to include a variety of DOK level problems and may include SAT style problems. |

| Code | *Pre-Assessment* | |
|---|---|---|
| **T, M**<br><br>**T, M, A** | ● Teacher checks for prerequisite skills and prior knowledge via warm-up and questioning activities, such as factoring and graphing functions<br>● Prerequisite knowledge will be reviewed as it is incorporated into topics both in class and on review assignments | |
| | Summary of Key Learning Events and Instruction<br>*Student success at transfer meaning and acquisition depends on…* | Progress Monitoring |
| T, M, A<br><br>M, A<br><br>T, M, A<br><br>T, M, A | ● Teacher will give warm up questions to lead into and review the concept of an imaginary number. Complex numbers will also be discussed, and teacher will model examples of simplifying and performing operations of addition, subtraction, multiplication, and division with complex numbers.<br>● Students will give ideas and examples of imaginary and complex numbers. Students will work as a class and then independently simplifying and performing mathematical operations with complex numbers. Individual students will put up answers to practice problems on the board.<br>● Teacher will use the overhead graphing calculator or smart board technology to facilitate class discovery of the effect the values a, h, and k have on the graphing form of a quadratic function ($y = a(x-h)^2+k$). Teacher will then use that discovery to introduce the key features of the graph of a parabola: vertex, axis of symmetry, direction of opening, max/min value, x- and y- intercept, domain and range. Teacher will model, with help from students, how to graph a parabola from that information. After mastery of this, teacher will build on the topic by modeling how to get any quadratic function into graphing form by the method of completing the square.<br>● Students will discover the properties of the graphing form of a quadratic function by observing the changes of the graph of a parabola when different values are inserted. | ● Warm up questions<br>● Monitoring class work through board work, group work, questioning, and walk-arounds<br>● Check for understanding via going over homework and medium such as reflections and exit tickets<br>● Class worksheets with direct teacher observation or self assessment<br>● Practice on whiteboard/chalkboard with direct teacher observation<br>● Kahoot quizzes with review questions and direct teacher observation<br>● Google form or other review assignments<br>● Homework assignments with direct teacher observation or self assessment<br>● Differentiate through purposeful or flexible grouping, use of diagrams and explanations to demonstrate understanding and active |

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| | Students will then work as a class and in pairs to find the key values of a parabola and to graph it. Student work will be put on the board as a way to review and monitor progress. | lessons involving discovery, scaffolding, jigsaw activities and use of hands-on manipulatives |
| M, A | ● Students will practice completing the square to get a quadratic function in graphing form by working in teacher created groups. | ● Summative assessments<br>Quizzes<br>Unit test |
| T, M, A | ● Teacher will introduce the graphs of polynomial functions by using the graphing calculator or smart board technology and student discovery of the nature of the graphs. Teacher will then model the steps to graphing polynomial functions by hand. | |
| M, A | ● Students will work independently or in pairs to discover the nature of the graphs of a polynomial function. Students will then practice graphing them by hand. Students will use the "Think-Pair-Share" method to compare their answers. | |
| T, M, A | ● Teacher will review long division with numbers to then model long division with polynomials. Synthetic division will then also be modeled. Teacher will make a connection to division of polynomials and whether or not a polynomial is a factor. The Remainder and Factor Theorems will be introduced and the teacher will lead students through the process of determining the remainder of the division of two polynomials, and whether a polynomial is a factor. | |
| M, A | ● Students will work as a class or in pairs to practice long and synthetic division and to use that division to determine the remainder. Students will apply the Factor Theorem to determine if a polynomial is a factor of a larger degree polynomial. Student work will be discussed orally as a group. | |
| T, M, A | ● Teacher will give a warm up question that reviews the domain of a rational function. This will lead into the concept of asymptotes for the graph of the rational function. Teacher will model the key components of vertical, horizontal, and slant asymptotes and how to use intercepts to determine the general shape of the graph. | |

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| A<br><br>M, A | ● Students will be placed in teacher created groups to graph rational functions. After students have mastered one or two, students will work independently on white boards or at the chalkboard graphing other rational functions.<br>● Students will produce an authentic graph of a unique rational functions<br><br>All Resources and materials must adhere to all New Milford Board of Education policies and regulations and are subject to New Milford Board of Education approval. Resources and materials must be researched and vetted by the writers and department heads prior to submission for approval.<br><br>● Textbook: Blitzer, Robert. Precalculus Second Edition, Upper Saddle River, NJ: Pearson, 2004.<br>● Supplemental activities from the textbook resources<br>● Teacher-made supplemental activities on applications, performance tasks, and chapter review<br>● Graphing calculator TI Emulator software.<br>● On-line resources such as YouTube, Khan Academy, Desmos, EdPuzzle, Kahoot, etc. | |

Unit 5: Exponential and Logarithmic Functions (Chapter 3)

ESTABLISHED GOALS

- **CC.9-12.F.IF.7e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **CC.9-12.F.BF.1** Write a function that describes a relationship between two quantities.*
- **CC.9-12.F.BF.5** (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents
- **CCSS.Math.Practice.MP 2** Reason abstractly and quantitatively.
- **CCSS.Math.Practice.MP 5** Use appropriate tools strategically.
- **CCSS.Math.Practice.MP 6** Attend to precision

***Transfer***

*Students will be able to independently use their learning to…*

- Analyze a problem and apply the appropriate techniques to simplifying expressions
- Reason abstractly
- Justify their reasoning or understanding by explaining
- Attend to precision when making mathematical statements

***Meaning***

UNDERSTANDINGS

*Students will understand that…*

- Exponential equations can be solved by getting a common base or by using logarithms.
- Logarithms are used to represent exponents, which could not be solved.
- Properties of logarithms relate to the properties of exponents.

ESSENTIAL QUESTIONS

*Students will keep considering…*

- What is the value of an exponential equation in the real world?
- What does a logarithm represent?
- How does the relationship between exponential and logarithmic functions help us?

***Acquisition***

*Students will know…*

- Exponential equations can be solved by getting a common base and by using logarithms

*Students will be skilled at…*

- Changing expressions to have the same base in order to solve exponential equations.

| | | |
|---|---|---|
| | ● A logarithm is a way to represent exponents<br>● There are  properties of logarithms that are related to the properties of exponents<br>● Common logarithms are logs with a base of 10 and can be used to solve exponential equations<br>● Natural logarithms have base e, which is a special irrational number used in exponential growth<br>● Key Terms:  exponent, exponential equation, base, logarithm, common logarithm, e, natural logarithm | ● Changing expressions from exponential form to logarithmic form, and vice-versa.<br>● Evaluating logarithmic expressions.<br>● Applying the properties of logarithm to solve logarithmic equations.<br>● Using common logarithms to solve exponential equations that do not have a common base.<br>● Identifying a natural logarithm as log base e. |

STAGE 2

| Code | Evaluative Criteria | Assessment Evidence |
|---|---|---|
| T, M, A | Evaluative Criteria consists of<br>● The steps are shown with work and explanation from photomath on the first set, using the notes on the second set, and completed independently by the student on the third set.<br>● Analysis contains reflection on the effectiveness of the photomath app<br>● Work is clear, correct, and neat. | PERFORMANCE TASK(S):<br>*Students will show that they really understand evidence of…*<br>Goal: To utilize the Photomath app as a means to learn how to solve exponential equations<br>Role: Member of focus group for the programmers of the Photomath app<br>Audience: Programming department<br>Situation: Students are given 4 exponential equations to first solve using the Photomath app. They are to write the steps down, then use those examples to solve 4 new equations. After that they will attempt to solve 4 additional equations without the use of any notes of examples. They will then give input to the programmers about the effectiveness of the directions given in the app.<br>Product: Students will present their work and an analysis of the photomath app as related to this topic |

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| A, M<br><br>A, M<br><br>A, M<br><br>T, A, M | Evaluative Criteria consists of<br>● Is the correct method used to solve the problem<br>● Is the analysis of the problem accurate<br>● Is the mathematics completed correctly<br>● Are students using correct mathematical terminology | OTHER EVIDENCE:<br>● Alternative assessment projects such as posters, drawings, pictures and real world applications<br>● Review of standardized test questions to prep students for the challenge of the SAT and college placement tests<br>● Performance tasks modeling real world and application problems<br>● Quizzes<br>● Unit Test - to include a variety of DOK level problems and may include SAT style problems. |

| Code | *Pre-Assessment* | |
|---|---|---|
| **T, M**<br><br>**T, M, A** | ● Teacher checks for prerequisite skills and prior knowledge via warm-up and questioning activities, such as properties of exponents, and solving various equations<br>● Prerequisite knowledge will be reviewed as it is incorporated into topics both in class and on review assignments | |
| | Summary of Key Learning Events and Instruction<br>*Student success at transfer meaning and acquisition depends on…* | Progress Monitoring |
| T, M | ● Teacher will use independent/guided practice via supplemental worksheets to review simplifying expressions with exponents. Teacher will walk around and monitor student progress, assist individual students, and will model examples when needed for the class. | ● Warm up questions |
| T, M, A | ● Students will work independently and in teacher created groups to complete practice problems that review exponents. Students will use “think-pair-share” to compare and discuss their answers. | ● Monitoring class work through board work, group work, questioning, and walk-arounds |
| T, M,A | ● Teacher will give a warm-up question on exponents as a way to lead in to solving exponential equations. Teacher will model different examples of exponential equations that have the same base and the process to solving them. | ● Check for understanding via going over homework and medium such as reflections and exit tickets |
| M, A | ● Students will complete problems on solving exponential equations. Students will volunteer their solutions, and will explain the process they used. | ● Class worksheets with direct teacher observation or self assessment |
| T, M | ● Teacher will have class graph the equation $y = 2^x$ and its inverse as a way of introducing the graph of an exponential equation and a logarithm. | ● Practice on whiteboard/chalkboard with direct teacher observation |
| T, M | ● Teacher will model how to solve and evaluate logarithmic equations and expressions by changing to exponential form and by applying the properties of logarithms. | ● Kahoot quizzes with review questions and direct teacher observation |
| T, M, A | ● Students will practice evaluating and solving logarithmic expressions and equations by various activities such as independent practice, board work, think-pair-share and/or use of white boards. | ● Google form or other review assignments<br>● Homework assignments with direct teacher observation or self assessment<br>● Differentiate through purposeful or flexible grouping, use of diagrams and explanations to demonstrate understanding and active |

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|---|---|---|
| | All Resources and materials must adhere to all New Milford Board of Education policies and regulations and are subject to New Milford Board of Education approval. Resources and materials must be researched and vetted by the writers and department heads prior to submission for approval.<br><br>● Textbook: Blitzer, Robert. Precalculus Second Edition, Upper Saddle River, NJ: Pearson, 2004.<br>● Supplemental activities from the textbook resources<br>● Teacher-made supplemental activities on applications, performance tasks, and chapter review<br>● Graphing calculator TI Emulator software.<br>● On-line resources such as YouTube, Khan Academy, Desmos, EdPuzzle, Kahoot, etc. | lessons involving discovery, scaffolding, jigsaw activities and use of hands-on manipulatives<br><br>● Summative assessments<br>Quizzes<br>Unit test |

Unit 6: Trigonometric Functions (chapter 4)

ESTABLISHED GOALS

- **CC.9-12.G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*
- **CC.9-12.F.TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **CC.9-12.F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- **CC.9-12.F.TF.3 (+)** Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x, and 2π - x in terms of their values for x, where x is any real number.
- **CC.9-12.F.TF.5** Choose trigonometric functions to model periodic phenomena

***Transfer***

*Students will be able to independently use their learning to…*

- Analyze a problem and apply the appropriate techniques to simplifying expressions
- Reason abstractly
- Justify their reasoning or understanding by explaining
- Attend to precision when making mathematical statements

***Meaning***

UNDERSTANDINGS

- Right triangle trigonometry has many uses and applications
- The unit circle can represent angles of any measure, in degrees or radians, and is cyclic.
- Trigonometric functions of specific angles relate to specific points and values on the unit circle.
- Graphs of the trigonometric functions are cyclic, with certain traits. Sine and cosine graphs produce “waves”.

ESSENTIAL QUESTIONS

- How can the use of right triangles, trigonometric functions, and the Pythagorean Theorem be used to solve real world problems?
- What is the unit circle and why is it important in trigonometry?
- Why do the graphs of trigonometric functions look the way they do?
- How do the graphs of sine and cosine apply to real life applications?
- What are the uses of inverse trigonometric functions?

***Acquisition***

*Students will know…*

*Students will be skilled at…*

| | | |
|---|---|---|
| with specified amplitude, frequency, and midline.*<br>• **CC.9-12.F.TF.6 (+)** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.<br>• **CCSS.Math.Practice.MP 2** Reason abstractly and quantitatively.<br>• **CCSS.Math.Practice.MP 5** Use appropriate tools strategically.<br>• **CCSS.Math.Practice.MP 6** Attend to precision<br>• | • The Pythagorean Theorem is a way to solve for a missing side in a right triangle, when given the other two sides<br>• The six trigonometric functions: sine, cosine, tangent, cosecant, secant, and cotangent and the relationships to the sides in a right triangle<br>• Radian measures express angles in terms of the arc length on the unit circle<br>• The special angles,points, and trigonometric values on the unit circle<br>• Reference angles help to find trig values in the second, third and fourth quadrants<br>• The methods to graphing sine, cosine, tangent, cosecant, and secant, and their general behaviors and critical values<br>• Inverse trigonometric functions can be used to solve for missing angles and have restrictions on their ranges in order to maintain them as a function<br>• Compositions of trigonometric and inverse functions<br>• Key Terms: Pythagorean Theorem, sine, cosine, tangent, cosecant, secant, cotangent, radian, degree, coterminal, complementary, supplementary, unit circle, angle in standard position, cyclic, reference angle, amplitude, period, phase shift, vertical shift, inverse trigonometric functions | • Using the Pythagorean Theorem and right triangle trigonometry to solve right triangles<br>• Defining the six trigonometric functions<br>• Constructing a unit circle and identify angles in both degree and radian measures<br>• Converting degrees to radians (and vice versa)<br>• Identifying specific points on the unit circle<br>• Defining the trigonometric functions as related to the x and y coordinate and radius on the unit circle<br>• Using reference angles and definitions of the trigonometric functions to find the specific values on the unit circle.<br>• Filling in the trigonometric table for values of special and quadrantile angles<br>• Graphing Sine, Cosine, Tangent, Cosecant, and Secant functions, and identify special characteristics such as amplitude, period, phase shift, and vertical shift<br>• Identifying the domain and range of inverse trigonometric functions<br>• Finding the exact values of inverse trigonometric functions and composite trigonometric functions<br>• Using graphing calculators to check graphs of trigonometric values, and to find approximate solutions to problems |

| Code | Evaluative Criteria | Assessment Evidence |
|---|---|---|
| T, M, A | Further information:<br>Evaluative Criteria consists of<br>● A completed product with correct work clearly shown<br>● Analysis of which trigonometric functions should be used for each problem<br>● Answers make sense in the context of the problem | PERFORMANCE TASK(S):<br>Performance Task #1:<br>Goal: To use right triangle trigonometry to solve real-world application problems<br>Role: Surveyor<br>Audience: Land development company<br>Situation: Given various situations, you are to calculate unknown distances to report back to the land development company for construction purposes<br>Product: Calculated distances with work shown |
| T, M, A | Evaluative Criteria consists of<br>● A personal biorhythm that is neat, organized, and well presented<br>● Correct calculations of age in days, and starting, ending, and critical values of each sine curve<br>● A key is provided for which graph is intellectual, emotional, and physical<br>● Argument of expected performance is accurate and supported by the graphs | Performance Task #2:<br>Goal: To calculate your personal biorhythm chart for the current month.<br>Role: Social scientist<br>Audience: School staff<br>Situation: You are to convince school staff whether or not your personal academic performance will be stronger or weaker based on your biorhythm<br>Product: Your completed biorhythm |

| | | |
|---|---|---|
| A, M<br><br>A, M<br><br>A, M<br><br>T, M, A | Evaluative Criteria consists of<br><br>● Is the correct method used to solve the problem<br><br>● Is the analysis of the problem accurate<br><br>● Is the mathematics completed correctly<br><br>● Are students using correct mathematical terminology | OTHER EVIDENCE:<br><br>● Alternative assessment projects such as posters, drawings, pictures and real world applications<br><br>● Review of standardized test questions to prep students for the challenge of the SAT and college placement tests<br><br>● Performance tasks modeling real world and application problems<br><br>● Quizzes<br><br>● Unit Test - to include a variety of DOK level problems and may include SAT style problems. |

| Code | *Pre-Assessment* | |
|---|---|---|
| **T, M**<br><br>**T, M, A** | ● Teacher checks for prerequisite skills and prior knowledge via warm-up and questioning activities, such as properties of right triangles and basics of graphing<br>● Prerequisite knowledge will be reviewed as it is incorporated into topics both in class and on review assignments | |
| | Summary of Key Learning Events and Instruction<br>*Student success at transfer meaning and acquisition depends on…* | Progress Monitoring |
| T, M, A | ● Teacher will review the Pythagorean Theorem and right triangle trigonometry. Teacher will give review and practice problems as classwork on finding missing sides and angles. Lesson will lead into the introduction of the three reciprocal trigonometric functions, and applications of trigonometry will be discussed. | ● Warm up questions<br>● Monitoring class work through board work, group work, questioning, and walk-arounds |
| T, M, A | ● Students practice solving right triangles using trigonometry by working in teacher created groups. Students will also identify the values of the reciprocal functions, and will use their calculators to find specific values. | ● Check for understanding via going over homework and medium such as reflections and exit tickets |
| T, M, A | ● Teacher will introduce the concept of the Unit Circle by first discussing radian measure as a representation of the length of the arc on the circle. Teacher will lead class through the discovery of the relationship between degrees and radians and how to convert degrees to radians and radians to degrees. | ● Class worksheets with direct teacher observation or self assessment<br>● Practice on whiteboard/chalkboard with direct teacher observation |
| M, A | ● Students will work independently on changing measures from degrees to radians and radians to degrees. Students may collaborate with a partner on their solutions. | ● Kahoot quizzes with review questions and direct teacher observation<br>● Google form or other review assignments |
| T, M, A | ● Teacher will continue to demonstrate the relations on the Unit Circle to points on the circle and angle measures. After a review of special right triangles, teacher will model how to find specific points given particular reference angles. | ● Homework assignments with direct teacher observation or self assessment |
| M, A | ● Students will complete the Unit Circle chart with specific degree measure, radian measure, and the coordinate of | ● Differentiate through purposeful or flexible grouping, use of diagrams and explanations to demonstrate understanding and active |

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| | the associated points. Students will then use the Unit Circle and reference angles to fill in the trigonometric table. | lessons involving discovery, scaffolding, jigsaw activities and use of hands-on manipulatives |
| M, A | ● Teacher will prepare materials (garland, laminated color coded cards with degree measures, radian measures, and coordinates of points) for the Unit Circle activity where students physically construct a model of the Unit Circle. | ● Summative assessments<br>Quizzes<br>Unit test |
| M, A | ● Students will work cooperatively as a group to construct the Unit Circle in the rotunda using garland and laminated values on the circle. Students will then play the “move it” game where they must move to a specific value on the circle. | |
| T, M, A | ● Teacher will review the relationships of trigonometric functions in right triangles and then show the connection with the x, y, and r values of the Unit Circle. Lesson will later lead into applications of the trigonometric functions to any point in the coordinate plane, which the teacher will model and explain. | |
| T, M, A | ● Students will apply the definitions of the trigonometric functions to the Unit Circle. Students will then find the values of the trigonometric functions at any value. | |
| T, M, A | ● With help of the graphing calculator, teacher will lead class through graphing the sine and cosine curves. Discussion on the general shape of the curves, their periodic behavior, and their amplitude, period, phase shift and vertical shift will occur. Teacher will lead class through examples on how to graph sine and cosine functions. | |
| M, A | ● Students will work at the board to practice graphing sine and cosine functions, identifying the amplitude, period, phase shift and vertical shift. | |
| M, A | ● Teacher will have class make a t-table to graph the tangent curve. The general shape of the curve, and its period will be discussed. Teacher will then model how to graph the secant and cosecant functions by using the sine and cosine graphs as “helpers”. | |
| M, A | ● Students will graph y=tanx as well as various cosecant and secant curves by working in teacher created groups. | |

| | | |
|---|---|---|
| T, M, A<br><br>M, A<br><br>T, M, A | ● Teacher will model how to use the graphing calculator and trigonometric table to find approximated and exact values of inverse trigonometric functions. Teacher will also explain how to find composite trigonometric values.<br>● Students will work in pairs to find inverse and composite trigonometric functions using their calculators and the trigonometric table.<br>● Teacher will determine cooperative groups for various activities during this unit<br><br>All Resources and materials must adhere to all New Milford Board of Education policies and regulations and are subject to New Milford Board of Education approval. Resources and materials must be researched and vetted by the writers and department heads prior to submission for approval.<br><br>● Textbook: Blitzer, Robert. Precalculus Second Edition, Upper Saddle River, NJ: Pearson, 2004.<br>● Supplemental activities from the textbook resources<br>● Teacher-made supplemental activities on applications, performance tasks, and chapter review<br>● Graphing calculator TI Emulator software.<br>● On-line resources such as YouTube, Khan Academy, Desmos, EdPuzzle, Kahoot, etc. | |

Unit 7: Law of Sines and Cosines (Chapter 6)

<table>
<tr><td rowspan="6">ESTABLISHED GOALS<br>● CC.9-12.G.SRT.11 (+)Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).<br>● CC.9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities<br>● CCSS.Math.Practice.MP 2 Reason abstractly and quantitatively.<br>● CCSS.Math.Practice.MP 5 Use appropriate tools strategically.<br>● CCSS.Math.Practice.MP 6 Attend to precision</td><td colspan="2">Transfer</td></tr>
<tr><td colspan="2">Students will be able to independently use their learning to…<br>● Analyze a problem and apply the appropriate techniques to simplifying expressions<br>● Reason abstractly<br>● Justify their reasoning or understanding by explaining<br>● Attend to precision when making mathematical statements</td></tr>
<tr><td colspan="2">Meaning</td></tr>
<tr><td>UNDERSTANDINGS<br>Students will understand that…<br>● The Law of Sines and Law of Cosines apply to non-right triangles and can be used to find missing lengths or angles</td><td>ESSENTIAL QUESTIONS<br>Students will keep considering…<br>● How can the use of trigonometric functions be extended to solve word problems and triangles with no right angles?</td></tr>
<tr><td colspan="2">Acquisition</td></tr>
<tr><td>Students will know…<br>● The Law of Sines is used to solve an oblique triangle where a side and angle opposite that side are given<br>● The ambiguous case of the Law of Sines occurs with the“Agles-Side-Side” case, and can result in zero, one or two triangle solutions</td><td>Students will be skilled at…<br>● Solving a triangle for missing sides or angles using the Law of Sines and the Law of Cosines<br>● Applying the ambiguous case of the Law of Sines to determine if there are no, one or two possible triangles.</td></tr>
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| | ● The Law of Cosines is used to solve an oblique triangle when 3 sides or 2 sides and an included angle are given<br>● Real world applications for the use of the Law of Sines and Law of Cosines<br>● Key Terms: Law of Sines, Law of Cosines, oblique triangle, ambiguous case | |

STAGE 2

| Code | Evaluative Criteria | Assessment Evidence |
|---|---|---|
| T, M, A | Evaluative Criteria consists of<br><br>● Presentation is neat, clear, with appropriate work shown<br>● Correct formulas are used for each problem, and solutions are also correct<br>● Solutions make sense in context of the problem | PERFORMANCE TASK(S):<br>*Students will show that they really understand evidence of…*<br><br>Goal: To find unknown values in specific real-world situations<br>Role: Surveyor<br>Audience: Manager of a development company<br>Situation: Given various situations, use the Laws of Sines and Cosines to calculate values that are otherwise un-measurable (example – calculate the distance between two landmarks that have a lake between them).<br>Product: Presentation (poster, slides, etc) of calculated distances with solutions shown |

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| A, M<br><br>A, M<br><br>A, M<br><br>T, A, M | Evaluative Criteria consists of<br><br>● Is the correct method used to solve the problem<br><br>● Is the analysis of the problem accurate<br><br>● Is the mathematics completed correctly<br><br>● Are students using correct mathematical terminology | OTHER EVIDENCE:<br>*Students will show they have achieved Stage 1 goals by…*<br><br>● Alternative assessment projects such as posters, drawings, pictures and real world applications<br><br>● Review of standardized test questions to prep students for the challenge of the SAT and college placement tests<br><br>● Performance tasks modeling real world and application problems<br><br>● Quizzes<br><br>● Unit Test - to include a variety of DOK level problems and may include SAT style problems. |

| Code | *Pre-Assessment* | |
|---|---|---|
| **T, M**<br><br>**T, M, A** | ● Teacher checks for prerequisite skills and prior knowledge via warm-up and questioning activities, such as properties of triangles, order of operations, and cross multiplication<br>● Prerequisite knowledge will be reviewed as it is incorporated into topics both in class and on review assignments | |
| | Summary of Key Learning Events and Instruction<br>*Student success at transfer meaning and acquisition depends on…* | Progress Monitoring |
| T, A, M<br><br>A, M<br><br>A, M<br><br>A, M<br><br>T, A, M<br><br>A, M<br><br>T, A, M | ● Teacher will introduce Law of Sines by having students solve a right triangle. Discussion will take place about solving a non right triangle, and teacher will give the formula for the Law of Sines and model some examples. Teacher will explain the cases when the Law of Sines can be used (AAS, ASA, SSA).<br>● Students will work as a whole group to practice some examples on the Law of Sines (non ambiguous case).<br>● Teacher will go over the ambiguous case for the Law of Sines and demonstrate why there are possibly no triangles, 1 triangle, or 2 triangles. Teacher will model examples with no triangle and 2 triangles.<br>● Students will practice examples of the ambiguous case of the Law of Sines by working in small teacher created groups.<br>● Teacher will give the formula for the Law of Cosines and discuss when to use it (SSS, SAS cases). Teacher will model an example where a side should be found first, and then one where an angle should be found first.<br>● Students will solve triangles using the Law of Cosines, and will compare their answers with a partner.<br>● Students will work independently to complete the performance task related to applications with the Law of Sines and Law of Cosines. | ● Warm up questions<br><br>● Monitoring class work through board work, group work, questioning, and walk-arounds<br><br>● Check for understanding via going over homework and medium such as reflections and exit tickets<br><br>● Class worksheets with direct teacher observation or self assessment<br><br>● Practice on whiteboard/chalkboard with direct teacher observation<br><br>● Kahoot quizzes with review questions and direct teacher observation<br><br>● Google form or other review assignments<br><br>● Homework assignments with direct teacher observation or self assessment<br><br>● Differentiate through purposeful or flexible grouping, use of diagrams and explanations to demonstrate understanding and active |

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| | All Resources and materials must adhere to all New Milford Board of Education policies and regulations and are subject to New Milford Board of Education approval. Resources and materials must be researched and vetted by the writers and department heads prior to submission for approval.<br><br>● Textbook: Blitzer, Robert. Precalculus Second Edition, Upper Saddle River, NJ: Pearson, 2004.<br>● Supplemental activities from the textbook resources<br>● Teacher-made supplemental activities on applications, performance tasks, and chapter review<br>● Graphing calculator TI Emulator software.<br>● On-line resources such as YouTube, Khan Academy, Desmos, EdPuzzle, Kahoot, etc. | lessons involving discovery, scaffolding, jigsaw activities and use of hands-on manipulatives<br><br>● Summative assessments<br>Quizzes<br>Unit test |