

[Teachers Companion Documents.zip](#)

## Number and Quantity

### The Real Number System

A2: N-RN

#### A. Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define  $5^{1/3}$  to be the cube root of 5 because we want  $(5^{1/3})^3 = 5^{(1/3)3}$  to hold, so  $(5^{1/3})^3$  must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

### Quantities\*

A2: N-Q

#### A. Reason quantitatively and use units to solve problems.

2. Define appropriate quantities for the purpose of descriptive modeling.

### The Complex Number System

A2: N-CN

#### A. Perform arithmetic operations with complex numbers.

1. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
2. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

#### C. Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.

## Algebra

### Seeing Structure in Expressions

A2: A-SSE

#### A. Interpret the structure of expressions.

2. Use the structure of an expression to identify ways to rewrite it. *For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .*

#### B. Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
  - c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression  $1.15^t$  can be rewritten as  $(1.15^{1/12})^{12t} \approx 1.012^{12t}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
4. Apply the formula for the sum of a finite geometric series (when the common ratio is not 1) to solve problems. *For example, calculate mortgage payments.* ★

### Arithmetic with Polynomials and Rational Expressions

A2: A-APR

#### B. Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**C. Use polynomial identities to solve problems.**

4. Use polynomial identities to describe numerical relationships. *For example, the polynomial identity  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  can be used to generate Pythagorean triples.*

**D. Rewrite rational expressions.**

6. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

**Creating Equations\***

**A2: A-CED**

**A. Create equations that describe numbers or relationships.**

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

**Reasoning with Equations and Inequalities**

**A2: A-REI**

**A. Understand solving equations as a process of reasoning and explain the reasoning.**

1. Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

**B. Solve equations and inequalities in one variable.**

4. Solve quadratic equations in one variable.
  - b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .

**C. Solve systems of equations.**

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), limited to systems of at most three equations and three variables. With graphic solutions, systems are limited to two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .*

**D. Represent and solve equations and inequalities graphically.**

11. Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★

**Functions**

**Interpreting Functions**

**A2: F-IF**

**B. Interpret functions that arise in applications in terms of the context.**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★*

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

**C. Analyze functions using different representations.**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
- b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ ,  $y = (1.2)^{t/10}$ , and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, determine which has the larger maximum.*

**Building Functions**

**A2: F-BF**

**A. Build a function that models a relationship between two quantities.**

1. Write a function that describes a relationship between two quantities. ★
- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★

**B. Build new functions from existing functions.**

3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
- a. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. *For example,  $f(x) = 2x^3$  or  $f(x) = (x+1)/(x-1)$  for  $x \neq 1$ .*

**Linear, Quadratic, and Exponential Models★**

**A2: F-LE**

**A. Construct and compare linear, quadratic, and exponential models and solve problems.**

2. Given a graph, a description of a relationship, or two input-output pairs (include reading these from a table), construct linear and exponential functions, including arithmetic and geometric sequences, to solve multi-step problems.

4. For exponential models, express as a logarithm the solution to  $a b^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

**B. Interpret expressions for functions in terms of the situation they model.**

5. Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.

**Trigonometric Functions**

**A2: F-TF**

**A. Extend the domain of trigonometric functions using the unit circle.**

- Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

**B. Model periodic phenomena with trigonometric functions.**

- Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.\*

**C. Prove and apply trigonometric identities.**

- Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant.

**Statistics and Probability\***

**Interpreting Categorical and Quantitative Data**

**A2: S-ID**

**A. Summarize, represent, and interpret data on a single count or measurement variable.**

- Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

**B. Summarize, represent, and interpret data on two categorical and quantitative variables.**

- Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
  - Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize exponential models.*

**Making Inferences and Justifying Conclusions**

**A2: S-IC**

**A. Understand and evaluate random processes underlying statistical experiments.**

- Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

**B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**

- Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
- Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

6. Evaluate reports based on data.

\*Modeling Standards - Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards. The basic modeling cycle involves

1. identifying variables in the situation and selecting those that represent essential features,
2. formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables,
3. analyzing and performing operations on these relationships to draw conclusions,
4. interpreting the results of the mathematics in terms of the original situation,
5. validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable,
6. reporting on the conclusions and the reasoning behind them.

Choices, assumptions, and approximations are present throughout this cycle.