NEW MILFORD PUBLIC SCHOOLS

New Milford, Connecticut



Advanced Algebra and Trigonometry

March 2023

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## **New Milford's Mission Statement**

The mission of the New Milford Public Schools, a collaborative partnership of students, educators, family and community, is to prepare each and every student to compete and excel in an ever-changing world, embrace challenges with vigor, respect and appreciate the worth of every human being, and contribute to society by providing effective instruction and dynamic curriculum, offering a wide range of valuable experiences, and inspiring students to pursue their dreams and aspirations.

## Advanced Algebra and Trigonometry

## Grade 11-12

Advanced Algebra and Trigonometry is a full year course designed for students who have completed a full year of Algebra 2. Topics in this course include a study of polynomial, trigonometric, exponential and logarithmic functions, graphing techniques, complex numbers, and topics in analytic geometry. A graphing calculator (TI-83+/TI-84+) is required for this course and is used throughout the year.

## Vision of a Graduate

Advanced Algebra and Trigonometry is a course that promotes problem solving, critical thinking, and a positive growth mindset. Throughout this course, students create a plan to solve a problem that follows a set procedure. Students will use effective reasoning to seek alternative and creative methods to find a solution. Students are expected to communicate verbally and through written expression how they arrived at their solution and what the solution means in the context of an application. Students will improve their skills through self-reflection and perseverance.

# Pacing Guide

Unit 1: Algebra Prerequisite Review	7-8 weeks
Unit 2: Graphs and Functions	4-6 weeks
Unit 3: Polynomial and Rational Functions	7-8 weeks
Unit 4: Exponential and Logarithmic Functions	3-4 weeks
Unit 5: Trigonometric Functions	7-8 weeks
Unit 6: Additional Topics in Trigonometry	1-2 weeks

ESTABLISHED GOALS	Tro	ansfer
CCSS.Math.Content.HSN.RN.A. 2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. CCSS.Math.Content.HSA.REI.A. 2 Solve simple rational and radical equations in one variable, and give examples showing how	Students will be able to independently use their learning Make sense of a problem by initiating a p Model with mathematics by using the ap Reason abstractly Justify reasoning or understanding by ex Attend to precision	to plan and persevere in solving propriate method kplaining techniques to solving
extraneous solutions may arise.	Me	eaning
CCSS.Math.Content.HSA.SSE.B .3. a Factor a quadratic expression to reveal the zeros of the function it defines. CCSS.Math.Content.HSA.CED.A .4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. CCSS.Math.Content.HSA.REI.B. 3 Solve linear equations and	<ul> <li>Students will understand that</li> <li>Exponents and radicals are related to the operation of addition and multiplication; a radical is the inverse of an exponent.</li> <li>Simplified radicals result in a smaller value under the radical while maintaining an exact value.</li> <li>Rationalizing the denominator eliminates radical expressions from the denominator.</li> <li>Radical expressions can be combined under the basic operations of addition, subtraction, multiplication, and division following a specific process.</li> </ul>	<ul> <li>Students will keep considering</li> <li>How are the properties of exponents related to the basic arithmetic operations?</li> <li>How do radicals relate to exponents?</li> <li>Why is it important to simplify radicals?</li> <li>Why is it necessary to rationalize the denominator?</li> <li>How do radical expressions relate to rational exponents?</li> <li>How are polynomial expressions combined using operations of addition, subtraction, and multiplication?</li> <li>How do you find the degree of a polynomial function?</li> </ul>
inequalities in one variable, including equations with coefficients represented by letters.	<ul> <li>Polynomials can be added, subtracted, and multiplied to make a more simplified expression.</li> <li>Polynomials can be broken up into products of more simplified terms by</li> </ul>	<ul> <li>What does the degree of a polynomial tell you about its related polynomial function?</li> <li>For a polynomial function, how are factors, zeros, and x-intercepts related?</li> <li>What is an extraneous solution for a rational</li> </ul>

<ul> <li>factoring.</li> <li>The degree of a polynomial is the greatest exponent among its monomial terms.</li> <li>Rational functions are a ratio of polynomial functions. If a rational function is in simplified form and the polynomial in the denominator is not constant, the graph of the rational function features asymptotic behavior. It looks quite different from the graph of either of its polynomial components.</li> <li>Multiplication and division of rational expressions uses much of what you know about multiplying and dividing fractions.</li> <li>Operations of rational expressions use much of what you know about multiplying and dividing fractions.</li> <li>Operations of rational expressions use much of what you know about operating with fractions. To add or subtract rational expressions must be solved by first multiplying each side by the least common denominator of the rational expressions. Doing this, however, can introduce extraneous solutions.</li> <li>Linear equations and inequalities can be solved following a specific process to give a solution.</li> <li>Literal equations are formulas with many variables and can be solved according to the same process as equations with one variable.</li> <li>Quadratic and other polynomial</li> </ul>	<ul> <li>function?</li> <li>Are a rational expression and its simplified form equivalent?</li> <li>What is the importance of following a specified order of operations?</li> <li>How are algebraic operations and notation used to simplify and solve equations and inequalities?</li> <li>How do literal equations apply to real-world situations?</li> <li>For a polynomial equation, how are factors and roots related?</li> </ul>
<ul> <li>Quadratic and other polynomial equations can be solved using factoring.</li> </ul>	

Acquisition	
Students will know	Students will be skilled at
<ul> <li>Students will know</li> <li>Properties of exponents</li> <li>Definition of nth root, radicand, index, and a principal root of a radical</li> <li>Steps and processes to simplify a radical expression.</li> <li>Properties for multiplying and dividing radical expressions.</li> <li>Steps and processes to "Rationalize the Denominator"</li> <li>Properties for adding and subtracting radical expressions.</li> <li>Steps and processes to multiply and divide binomial radical expressions.</li> <li>Steps and processes to multiply and divide binomial radical expressions.</li> <li>Methods and processes to simplify expressions with rational exponents.</li> <li>Degree of a monomial and polynomial</li> <li>Definition of an algebraic term Addition, subtraction, and multiplication processes of polynomials</li> <li>Steps and processes to factoring polynomials</li> <li>Restrictions on the domain of a rational expression.</li> <li>Steps and processes to simplifying, multiplying, and dividing rational expressions.</li> <li>Methods and processes to simplifying rational expressions.</li> </ul>	<ul> <li><i>Students will be skilled at</i></li> <li>Simplifying expressions using the rules of exponents</li> <li>Simplifying nth roots.</li> <li>Determining all real roots of a real number and the degree of a radical expression.</li> <li>Simplifying radical expressions.</li> <li>Multiplying and dividing radical expressions.</li> <li>Rationalizing the denominator of a radical expression.</li> <li>Adding and subtracting radical expressions.</li> <li>Multiplying and dividing binomial radical expressions.</li> <li>Identifying the degree of a monomial and polynomial</li> <li>Classifying a polynomial by the number of terms</li> <li>Performing the operations of Addition, subtraction, and multiplication of polynomials</li> <li>Factoring polynomial expressions</li> <li>Identifying values that are restricted from the domain of a rational expression.</li> <li>Writing a rational expression in simplest form.</li> <li>Multiplying and dividing rational expressions by factoring.</li> <li>Adding and subtracting rational expressions.</li> <li>Simplifying complex rational expressions.</li> </ul>
complex rational expressions.	Comparing values both on a number line
Process to solving rational equations.	and using inequality symbols.
<ul> <li>Comparing values (&gt;, &lt;, =).</li> </ul>	Finding the absolute value of a number
<ul> <li>The absolute value of a number.</li> <li>The process for evaluating and</li> </ul>	<ul> <li>Simplifying and evaluate algebraic expressions</li> </ul>

<ul> <li>simplifying algebraic expressions and the specific order of operations.</li> <li>How to solve a linear and literal equation</li> <li>Steps to solving inequalities and absolute value equations and inequalities and how to graph solutions on a number line.</li> <li>Methods and processes to solving a polynomial equation</li> </ul>	<ul> <li>Solving linear, literal, and absolute value equations and identifying those that have no solution.</li> <li>Solving inequalities, compound inequalities, and absolute value inequalities according to specified processes.</li> <li>Solving polynomial equations by factoring</li> </ul>
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Code	Evaluative Criteria	Assessment Evidence
		PERFORMANCE TASK(S): Students will show that they really understand evidence of
T, M, A	Scoring Rubric used to evaluate successful understanding of the process and criteria for a desired outcome.	<u>Goal</u> : To identify correct and incorrect steps for solving linear/rational/quadratic equations <u>Role</u> : Teacher <u>Audience</u> : Student who solved the problem <u>Situation</u> : Students are given a problem set with specific steps shown as a solution. Students then identify if each step is correct or incorrect and explain why. <u>Product</u> : Corrected problem to include feedback and explanation. <u>Standard for Success</u> : rubric based on understanding, accuracy, communication of results, presentation of evidence to support claim. <u>To Differentiate</u> : Allow students to choose from problems at a variety of difficulty levels.

		OTHER EVIDENCE:
		Students will show they have achieved Stage 1 goals by
М	Thorough understanding of identifying values that are restricted from the domain, simplifying a rational expression, types of polynomials.	<ul> <li>Alternative assessment projects such as "find the mistakes", explain the process, posters, and real world applications</li> <li>Review of standardized test questions to prep students for the challenge of the SAT and ACT exams</li> <li>Quizzes</li> </ul>
T, M, A	Thorough understanding of steps and processes to simplify, multiply, divide, add, subtract, and solve equations.	<ul> <li>UNIT Test</li> <li>"Do Now" questions/opening</li> <li>Activities</li> <li>Questioning</li> </ul>
T, M, A	Accurate application of content/process to arrive at the correct mathematical solution.	<ul> <li>Self-assessment</li> <li>Smartboard activities, (Kahoot, Quizlet, etc.)</li> </ul>
T, M, A	Selection of evidence that is relevant to content and standardized test processes.	

### stage 3

Code	Pre-Assessment
	<ul> <li>Teacher checks for prerequisite and prior knowledge via warm-ups and entrance tickets</li> </ul>
Μ	Questioning activities, such as basic problems with exponents and radicals.
	• As the lessons progress, students can also be given questions such as "Find the mistakes"
	Warm-ups and skill checks contain review of previous material during the unit to ensure retention and
	mastery, and check on vertical alignment with prior curriculum.

	Summary of Key Learning Events and Instruction	Progress Monitoring
	Student success at transfer meaning and acquisition depends on	
		<ul> <li>Monitoring class work through board work,</li> </ul>
M, A	Teacher checks for prior knowledge using common	group work, questioning, warm-ups, and
	formative assessment (pre-test) on properties of	walk-arounds.
	exponents and solving polynomial equations.	<ul> <li>Homework check to assess common errors to</li> </ul>
T, M, A	• Students will work independently on a pre-test for the	inform future instruction.
	properties of exponential expressions.	Check prerequisite knowledge throughout the
M, A	<ul> <li>Teacher models real roots by writing y<sup>2</sup> = 64 on the board to show the number of real nth roots.</li> </ul>	unit using warm-up problems and questioning activities.
M. A	• Teacher reviews the perfect square factors, perfect cube	Differentiate through purposeful or flexible
,	factors, perfect fourth root factors, etc. to explain the steps	grouping, use of diagrams and explanations to
	for simplifying radical expressions. Teacher reiterates the	demonstrate understanding and active lessons
	importance of factoring out the greatest of these types of	involving discovery, scaffolding, jigsaw activities
	factors first.	and use of hands-on manipulatives
M, A	<ul> <li>Students work independently to simplify radicals.</li> </ul>	• Check for understanding: board and whiteboard
M, A	• Teacher models the properties and steps for multiplying	activities, or reflections and exit tickets.
	and dividing radical expressions.	<ul> <li>Strategic Questioning: Ask students</li> </ul>
M, A	• Teacher introduces the concept of "Rationalizing the	higher-order questions such as "how" and "why,"
	Denominator" as an alternate method to dividing radical	so the teacher can discern the level and extent
	expressions when the denominator contains a radical.	of the students' understanding.
	reacher defines like radicals to model adding and	
	the need for students to first simplify the radical	
	expression they want to add or subtract	
N4 A	<ul> <li>Students practice the steps to multiplying and dividing</li> </ul>	
IVI, A	radical expressions.	
M, A	• Teacher makes a connection using the FOIL method for	
	multiplying binomials to multiplying binomial radical	
	expressions.	
M	Ieacher models the addition, subtraction, and     multiplication of polynomials	
N4 T	multiplication of polynomials.	
IVI, I	• Students will verbally explain the process of adding,	
	evolain what FOIL means in the multiplication of	
Μ	Teacher explains factoring of a polynomial expression	

	hearing with CCF and exemples on day and then the	
	beginning with GCF and grouping on day one, then the	
	difference of squares and the sum and difference of	
	cubes, and lastly trinomials.	
Т	<ul> <li>Teacher allows students several opportunities for</li> </ul>	
	independent practice and teacher-created groups	
	throughout this topic. Supplemental worksheets and board	
	problems should be used to assess mastery of this	
	concept.	
T.A	<ul> <li>Students will use smartboard to practice factoring</li> </ul>	
- ,	polynomial expressions. Students will work in pairs on a	
	mixed review assessment on factoring to explain which	
	method of factoring should be used	
М	<ul> <li>Teacher uses flow charts to help students determine</li> </ul>	
	which method of factoring should be used to factor a	
	nolynomial	
М	<ul> <li>Teacher highlights for students that no matter what a</li> </ul>	
141	polynomial looks like the process of factoring always	
	starts with factoring out a GCE if possible	
M	<ul> <li>Teacher activates prior knowledge via pre-assessment</li> </ul>	
171	• reacher activates phot knowledge via pre-assessment	
	simplifying exponential expressions, factoring	
	simplifying exponential expressions, factoring	
N4 A	Polyhomiais, and solving polyhomiai equations.	
IVI,A	<ul> <li>Students will work independently and as a class factoring polynomial expressions and solving polynomial equations</li> </ul>	
N.4	polynomial expressions and solving polynomial equations.	
IVI	• reacher introduces the concept of simplifying rational	
N.4 A	expressions as being dependent on factoring polynomials.	
IVI, A	<ul> <li>Students will use the white boards to practice simplifying retioned expressions</li> </ul>	
N.4	rational expressions.	
IVI	reacher models multiplying and dividing rational	
N4 A	expressions.	
M, A	<ul> <li>Students will explain in writing whether or not terms can</li> </ul>	
	be canceled in a rational expression and why.	
M, A	<ul> <li>Students will recite the meaning of "Keep-Change-Flip" for</li> </ul>	
M	Ieacher makes connections to the similarities in the	
	process used in adding/subtracting rational expressions	
	versus basic fractions.	

М	• Teacher invites a volunteer to write the steps for adding a	
	fraction similar to $\frac{1}{2} + \frac{5}{8}$ on the board.	
M, A	<ul> <li>Students will verbally explain the process of finding the</li> </ul>	
	least common denominator (LCD) in order to add and	
	subtract rational expressions.	
M, A	<ul> <li>Students will practice simplifying complex rational</li> </ul>	
	expressions as a class and independently.	
M,A	<ul> <li>Students will solve rational equations by</li> </ul>	
	cross-multiplication or by setting the LCDs equal to each	
	other.	
M,A	<ul> <li>Teacher suggests changing a complex fraction to a</li> </ul>	
	division problem in the form of $(N) \div (D)$ where N and D	
	are the expressions in the numerator and denominator of	
	the complex fraction.	
Т, М,А	<ul> <li>Supplemental worksheets and board problems should be</li> </ul>	
	used to assess mastery of the process.	
M, A	<ul> <li>Teacher models examples evaluating expressions and</li> </ul>	
	solving equations.	
M, A	<ul> <li>Students will work independently practicing evaluating</li> </ul>	
	and simplifying algebraic expressions.	
Т, М	<ul> <li>Teacher uses TI Emulator software to demonstrate the</li> </ul>	
	process of using the graphing calculator to evaluate	
	expressions and to check solutions to equations.	
Т, М, А	<ul> <li>Students will practice using the graphing calculator to</li> </ul>	
	solve equations and to evaluate expressions.	
M, A	• leacher models steps to solving algebraic, literal, and	
	fractional equations by hand. Teacher also relates literal	
	equations to formulas and other real-world uses.	
М, А	<ul> <li>Students will work in small groups to solve equations that include basis fractional and literal types</li> </ul>	
<b>T</b> N4	Include basic, fractional, and literal types.	
Ι, ΙVΙ	<ul> <li>Students will describe the factoring methods for solving networking and practice this method in teacher</li> </ul>	
	assigned groups.	
	Resources:	

All Resources and materials must adhere to all New Milford Board of Education policies and regulations and are subject to New Milford Board of Education approval. Resources and materials must be researched and vetted by the writers and department heads prior to submission for approval.	
<ul> <li>Textbook: Blitzer, Robert. Precalculus Second Edition, Upper Saddle River, NJ: Pearson, 2004.</li> <li>Supplemental activities from the textbook resources</li> <li>Teacher-made supplemental activities on applications, performance tasks, and chapter review</li> <li>Graphing calculator TI Emulator software.</li> <li>On-line resources such as YouTube, Khan Academy, Desmos, EdPuzzle, Kahoot, etc.</li> </ul>	

UbD Template 2.0

UNIT 2: Graphs and Functions

ESTABLISHED GOALS	Transfer           Students will be able to independently use their learning to	
Include any national/state/or school goals (Power standards).		
CCSS.Math.Content.HSA.CED.A .2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	Make sense of a problem by initiating a plan and persevere in solving Model with mathematics by using the appropriate method Reason abstractly Justify reasoning or understanding by explaining techniques to solving Attend to precision	
CCSS.Math.Content.HSF.IF.A.1	M	eaning
one set (called the domain) to another set (called the range)	UNDERSTANDINGS Students will understand that	ESSENTIAL QUESTIONS Students will keep considering

<ul> <li>A line contains its own unique slope and y-intercept.</li> <li>The slope of a line represents the rate of change of that line.</li> <li>The slope of a line represents the rate of change of that line.</li> <li>Information about a line can be used to write an equation for that line.</li> <li>Information about a line can be used to write an equation for that line.</li> <li>Relations have specific characteristics that result in them being considered functions.</li> <li>CCSS.Math.Content.HSF.IF.A.2 Jse function notation, evaluate unctions for inputs in their domains, and interpret statements that use function totation in terms of a context.</li> <li>CCSS.Math.Content.HSF.IF.B.4</li> <li>For a function that models a elationship between two guantities, interpret key features of graphs and tables in terms of he quantities, and sketch graphs</li> </ul>		<ul> <li>What are the different forms of linear equations, and why is it useful to have them?</li> <li>What applications can be represented by linear equations?</li> <li>What are the similarities and differences between the graphs of linear functions and absolute value functions?</li> <li>Why is it important to represent the same relation or function using multiple formats?</li> <li>How are the characteristics of a linear equation?</li> <li>What are some real-world uses of inverse and composite functions?</li> <li>What does the parent function of various graphs look like, and how do values change the graph?</li> </ul>
verbal description of the	Acq	uisition
relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity . * CCSS.Math.Content.HSF.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For	<ul> <li>What the slope of a line represents and how to find it</li> <li>The distinction between relations and functions.</li> <li>How to find the domain and range of a function</li> <li>The processes to identify and evaluate functions</li> <li>The key features of graphs. To include: intercepts; intervals where the function is positive, or negative, increasing, or</li> </ul>	<ul> <li>Recognizing slope as a rate of change</li> <li>Identifying slopes of horizontal, vertical, parallel, and perpendicular lines</li> <li>Writing and graph equations in point-slope and slope-intercept form</li> <li>Determining the domain and range from a relation and represent the relation using a mapping diagram</li> <li>Deciding if a relation is a function when</li> <li>given a set of ordered pairs, a mapping, and a graph</li> </ul>

example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.	<ul> <li>decreasing; relative maximums and minimums; symmetries, and end behavior.</li> <li>What composition of functions is and how to apply it</li> <li>The steps to graphing functions from the parent function.</li> </ul>	<ul> <li>Evaluating a specific value given the equation or graph of a function (i.e., find f(3) given the graph of f(x))</li> <li>Identifying key features of graphs.</li> <li>Composing two functions</li> <li>Finding the inverse of a function and graphing it</li> <li>Determining key features of a graph (vertex, vertical and horizontal shifts, stretch or compression) based on transformations of its parent function.</li> </ul>
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Code	Evaluative Criteria	Assessment Evidence
	Further information:	PERFORMANCE TASK(S):
		Students will show that they really understand evidence of
T, M, A	Scoring Rubric used to evaluate successful understanding of the process and criteria for a desired outcome.	<u>Goal</u> : To find the line of best fit given real-world data <u>Role</u> : Financial consultant <u>Audience</u> : Business managers for various companies <u>Situation</u> : Given three different companies (photography, home improvement, and theater), the consultant is asked to provide a cost analysis from given data. <u>Product</u> : Calculated predictions with appropriate explanations <u>Standard for Success</u> : rubric based on understanding, accuracy, communication of results, presentation of evidence to support claim. <u>To Differentiate</u> : Allow students to choose from problems at a variety of difficulty levels.

		OTHER EVIDENCE:
		Students will show they have achieved Stage 1 goals by
Μ	Thorough understanding of identifying values that are restricted from the domain, simplifying a rational expression, types of polynomials.	<ul> <li>Alternative assessment projects such as "find the mistakes", explain the process, posters, and real world applications</li> <li>Review of standardized test questions to prep students for the challenge of the SAT and ACT exams</li> <li>Quizzes</li> </ul>
T, M, A	Thorough understanding of steps and processes to simplify, multiply, divide, add, subtract, and solve equations.	<ul> <li>UNIT Test</li> <li>"Do Now" questions/opening</li> <li>Activities</li> <li>Questioning</li> </ul>
T, M, A	Accurate application of content/process to arrive at the correct mathematical solution.	<ul> <li>Self-assessment</li> <li>Smartboard activities, (Kahoot, Quizlet, etc.)</li> </ul>
T, M, A	Selection of evidence that is relevant to content and standardized test processes.	

Code	Pre-Assessme	nt
М	<ul> <li>Teacher checks for prerequisite and prior knowledge via warm-ups and entrance tickets</li> <li>Questioning activities, such as basic problems with exponents and radicals.</li> <li>As the lessons progress, students can also be given questions such as "Find the mistakes"</li> <li>Warm-ups and skill checks contain review of previous material during the unit to ensure retention and mastery, and check on vertical alignment with prior curriculum.</li> </ul>	
	Summary of Key Learning Events and Instruction	Progress Monitoring
M, A	<ul> <li>Student success at transfer meaning and acquisition depends on</li> <li>Teacher activates prior learning by giving practice through homework, warm-ups, and entrance tickets to review linear equations and slope</li> </ul>	<ul> <li>Monitoring class work through board work, group work, questioning, warm-ups, and walk-arounds.</li> <li>Homework check to assess common errors to</li> </ul>
A	<ul> <li>Students review and practice writing and graphing linear equations.</li> </ul>	<ul> <li>inform future instruction.</li> <li>Check prerequisite knowledge throughout the</li> </ul>
M, A	<ul> <li>Teacher activates prior learning of graphing by giving warm-up exercises on graphing points in the coordinate</li> </ul>	unit using warm-up problems and questioning activities.

A T, M, A T, M, A T, A T, M, A T, M, A T, M, A	<ul> <li>plane and evaluating expressions.</li> <li>Teacher defines relation, domain,range, and function and models examples on how to identify these from given information (data sets, mapping, graph).</li> <li>Students will work independently to identify domain and range and determine if a relation is a function.</li> <li>Students will also evaluate functions for given values.</li> <li>Teacher discusses the real-world application of composition of functions and models the process of composing two functions into one new function.</li> <li>Students will complete a practice worksheet on composition of functions and will then compare and discuss their results with a partner.</li> <li>Students will use graphing technology to discover how graphs are related to their parent function and what causes them to shift vertically, horizontally, and to stretch or compress.</li> <li>Teacher provides an activity for students to discover what an inverse of a functions.</li> <li>Students will work collaboratively to discover the relationship between a function and its inverse and will complete a practice worksheet on find inverses of given functions.</li> </ul>	<ul> <li>Differentiate through purposeful or flexible grouping, use of diagrams and explanations to demonstrate understanding and active lessons involving discovery, scaffolding, jigsaw activities and use of hands-on manipulatives</li> <li>Check for understanding: board and whiteboard activities, or reflections and exit tickets.</li> <li>Strategic Questioning: Ask students higher-order questions such as "how" and "why," so the teacher can discern the level and extent of the students' understanding.</li> </ul>
	<ul> <li><u>Resources:</u> <ul> <li>All Resources and materials must adhere to all New Milford Board of Education policies and regulations and are subject to New Milford Board of Education approval. Resources and materials must be researched and vetted by the writers and department heads prior to submission for approval.</li> <li>Textbook: Blitzer, Robert. Precalculus Second Edition, Upper Saddle River, NJ: Pearson, 2004.</li> <li>Supplemental activities from the textbook resources</li> </ul> </li> </ul>	

<ul> <li>Teacher-made supplemental activities on applications, performance tasks, and chapter review</li> <li>Graphing calculator TI Emulator software.</li> <li>On-line resources such as YouTube, Khan Academy, Desmos, EdPuzzle, Kahoot, etc.</li> </ul>	
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UbD Template 2.0

### UNIT 3: Polynomial and Rational Functions

ESTABLISHED GOALS	Transfer	
CCSS.MATH.CONTENT.HSA.AP R.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by x - a is $p(a)$ , so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ .	Students will be able to independently use their learning to Make sense of a problem by initiating a plan and persevere in solving Model with mathematics by using the appropriate method Reason abstractly Justify reasoning or understanding by explaining techniques to solving Attend to precision	
CCSS.MATH.CONTENT.HSA.AP		
R.B.3 Identify zeros of	M	eaning
polynomials when suitable	UNDERSTANDINGS	ESSENTIAL QUESTIONS
factorizations are available, and	Students will understand that	Students will keep considering
use the zeros to construct a rough graph of the function defined by the polynomial.	<ul> <li>Polynomials can be added, subtracted, and multiplied to make a more simplified expression.</li> </ul>	<ul> <li>How are polynomial expressions combined using operations of addition, subtraction, and multiplication?</li> </ul>
CCSS.MATH.CONTENT.HSA.AP R.A.1 Understand that	<ul> <li>Polynomials can be broken up into products of more simplified terms by</li> </ul>	<ul> <li>Why does factoring "work" as a method of solving quadratic and polynomial equations?</li> </ul>
polynomials form a system	factoring.	What are some real-world applications that
analogous to the integers,	<ul> <li>Quadratic and other polynomial</li> </ul>	involve polynomial modeling?
namely, they are closed under the operations of addition,	equations can be solved using factoring, completing the square, and Quadratic	<ul> <li>Why do some functions have restricted values?</li> </ul>

subtraction, and multiplication; add, subtract, and multiply polynomials. CCSS.MATH.CONTENT.HSA.R EI.B.4.B Solve quadratic equations by inspection (e.g., for x 2= 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and	<ul> <li>Formula.</li> <li>Graphs of Polynomial functions can be used to find domain, range, and intercepts and to tell the nature of the function( increasing, decreasing, constant , maxima, minima)</li> <li>Rational functions are a ratio of polynomial functions. If a rational function is in simplified form and the polynomial in the denominator is not constant, the graph of the rational function features asymptotic behavior. It looks quite different from the graph of either of its polynomial components.</li> </ul>	<ul> <li>How do you find the degree of a polynomial function?</li> <li>What does the degree of a polynomial tell you about its related polynomial function?</li> <li>For a polynomial function, how are factors, zeros, and x-intercepts related?</li> <li>For a polynomial equation, how are factors and roots related?</li> <li>Writing a rational expression in simplest form.</li> <li>What kinds of asymptotes are possible for a rational function?</li> </ul>
b	Acq	luisition
	Students will know	Students will be skilled at
CCSS.MATH.CONTENT.HSA.R EI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line)	<ul> <li>Degree of a monomial and polynomial</li> <li>Definition of an algebraic term</li> <li>Addition, subtraction, and multiplication processes of polynomials</li> <li>Steps and processes to factoring polynomials</li> <li>Methods and processes to solving a polynomial equation</li> <li>A polynomial function is classified by degree.</li> <li>The degree of a polynomial determines the possible number of turning points in its graph and the end behavior of the graph.</li> <li>A turning point is a relative maximum or relative minimum of a polynomial function.</li> <li>What constitutes even vs. odd multiplicity when the function is in its algebraic form.</li> </ul>	<ul> <li>Identifying the degree of a monomial and polynomial</li> <li>Classifying a polynomial by the number of terms</li> <li>Performing the operations of Addition, subtraction, and multiplication of polynomials</li> <li>Factoring polynomial expressions</li> <li>Solving polynomial equations by factoring or graphing methods</li> <li>Graphing polynomials and Identifying intercepts, points of relative maxima and minima, intervals where the function is increasing, decreasing, or constant, as well as find specific values from the graph of a function</li> <li>Recognizing from a graph the key features of a polynomial such as the factors, zeros, relative minimums, relative maximums.</li> </ul>

<ul> <li>Steps and processes to simplifying, multiplying, and dividing rational expressions.</li> <li>Restrictions on the domain of a rational expression.</li> </ul>	<ul> <li>Identifying values that are restricted from the domain of a rational expression.</li> <li>Writing a rational expression in simplest form.</li> </ul>
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Code	Evaluative Criteria	Assessment Evidence
	Further information:	PERFORMANCE TASK(S):
		Students will show that they really understand evidence of
T, M, A	Scoring Rubric used to evaluate successful	
	understanding of the process and criteria for a	Goal: To apply the skills of polynomial functions in the design of
	desired outcome.	roller coaster rides.
		Role: Roller Coaster Engineer
		Audience : Amusement Park Manager
		Situation : Given three different polynomial functions that model
		roller coasters, the student is asked to graph each function, find the
		heights at different independent variables (time), and evaluate the
		function at a given independent variable.
		Product: Demonstration of a clear and in depth understanding of
		polynomial functions, such as sketching and analyzing graphs of
		polynomial functions, determining zeros of a polynomial function.
		and determining polynomial function behavior
		Standard for Success: rubric based on understanding accuracy
		communication of results, presentation of evidence to support
		claim
		To Differentiate: Provide different problems with different levels of
		difficulty from which students can choose

Μ	Thorough understanding of identifying values	OTHER EVIDENCE:
	that are restricted from the domain,	Students will show they have achieved Stage 1 goals by
	simplifying a rational expression, types of	
	polynomials.	<ul> <li>Alternative assessment projects such as "find the mistakes", explain the process, posters, and real world applications</li> </ul>
T, M, A	Thorough understanding of steps and processes to simplify, multiply, divide, add, subtract, and solve equations.	<ul> <li>Review of standardized test questions to prep students for the challenge of the SAT and ACT exams</li> <li>Quizzes</li> </ul>
		UNIT Test
T, M, A	Accurate application of content/process to	<ul> <li>"Do Now" questions/opening</li> </ul>
	arrive at the correct mathematical solution.	Activities
		Questioning
T, M, A	Selection of evidence that is relevant to content and standardized test processes.	<ul> <li>Self-assessment</li> <li>Smartboard activities, (Kahoot, Quizlet, etc.)</li> </ul>

Code	Pre-Assessme	nt
М	<ul> <li>Teacher checks for prerequisite and prior knowledge via warm-ups and entrance tickets</li> <li>Questioning activities, such as basic problems with exponents and radicals.</li> <li>As the lessons progress, students can also be given questions such as "Find the mistakes"</li> <li>Warm-ups and skill checks contain review of previous material during the unit to ensure retention and mastery and check on vertical alignment with prior curriculum</li> </ul>	
	Summary of Key Learning Events and Instruction	Progress Monitoring
M T M	<ul> <li>Student success at transfer meaning and acquisition depends on</li> <li>Teacher checks for prerequisite and prior knowledge via warm-ups on solving and graphing linear functions</li> <li>Students will work independently and as a class solving equations and graphing linear equations both manually and on graphing calculators.</li> <li>Teacher introduces the properties of exponents by using visual representations to what exponents mean (e.g., x<sup>2</sup> · x<sup>3</sup> = x5 since x<sup>2</sup> = x · x and x<sup>3</sup> = x · x · x giving us a result of 5 x's).</li> </ul>	<ul> <li>Monitoring class work through board work, group work, questioning, warm-ups, and walk-arounds.</li> <li>Homework check to assess common errors to inform future instruction.</li> <li>Check prerequisite knowledge throughout the unit using warm-up problems and questioning activities.</li> <li>Differentiate through purposeful or flexible grouping, use of diagrams and explanations to</li> </ul>

Т, М	Students will work independently simplifying exponential	demo
	expressions and then share results in a teacher created	involv
	pairing.	activit
Μ	<ul> <li>Teacher models the addition, subtraction, and</li> </ul>	Check
	multiplication of polynomials.	whitek
М, Т	<ul> <li>Students will verbally explain the process of adding,</li> </ul>	tickets
	subtracting, and multiplying polynomials. Students will	<ul> <li>Strate</li> </ul>
	explain what FOIL means in the multiplication of	highe
	polynomials.	"why,"
M	<ul> <li>leacher explains factoring of a polynomial expression</li> </ul>	exten
	beginning with GCF and grouping on day one, then the	
	difference of squares and the sum and difference of	
т	Cubes, and lastly innomials.	
I	<ul> <li>Teacher anows students several opportunities for independent practice and teacher created groups</li> </ul>	
	throughout this topic. Supplemental worksheets and hoard	
	problems should be used to assess mastery of this	
	concept.	
T.A	<ul> <li>Students will use smartboard to practice factoring</li> </ul>	
,	polynomial expressions. Students will work in pairs on a	
	mixed review assessment on factoring to explain which	
	method of factoring should be used.	
Μ	<ul> <li>Teacher uses flow charts to help students determine</li> </ul>	
	which method of factoring should be used to factor a	
	polynomial.	
T,M	• Students will describe the factoring methods for solving	
	polynomial equations and practice this method in teacher	
N 4	assigned groups.	
IVI	<ul> <li>Teacher highlights for students that no matter what a networking leake like, the presence of featuring elways</li> </ul>	
	starts with factoring out a CCE if possible. Teacher	
	illustrates a method for solving polynomial equations by	
	factoring by hand and by graphing calculators	
МА	<ul> <li>Students will solve polynomial equations having a degree</li> </ul>	
,	areater than two by entering the linear portion in Y1 in	
	their graphing calculators and the rest of the equation in	
	Y2 of their graphing calculators. Students will then use the	

demonstrate understanding and active lessons involving discovery, scaffolding, jigsaw activities and use of hands-on manipulatives

- Check for understanding: board and whiteboard activities, or reflections and exit tickets.
- Strategic Questioning: Ask students higher-order questions such as "how" and "why," so the teacher can discern the level and extent of the students' understanding.

	intersect feature to find the x-values at that point of intersection.	
Μ	<ul> <li>Teacher activates prior knowledge via pre-assessment worksheets on the adding/subtracting of rational numbers, simplifying exponential expressions, factoring</li> </ul>	
	polynomials, and solving polynomial equations.	
Μ	<ul> <li>Teacher models how to identify the asymptotes and holes to graph a rational function.</li> </ul>	
M, A	<ul> <li>Students will work independently and as a class graphing rational functions.</li> </ul>	
T, M, A	<ul> <li>Supplemental worksheets and board problems should be used to assess mastery of the process in graphing a rational function.</li> </ul>	
	Resources:	
	All Resources and materials must adhere to all New Milford Board of Education policies and regulations and are subject to New Milford Board of Education approval. Resources and materials must be researched and vetted by the writers and department heads prior to submission for approval.	
	<ul> <li>Textbook: Blitzer, Robert. Precalculus Second Edition, Upper Saddle River, NJ: Pearson, 2004.</li> <li>Supplemental activities from the textbook resources</li> <li>Teacher-made supplemental activities on applications,</li> </ul>	
	<ul> <li>performance tasks, and chapter review</li> <li>Graphing calculator TI Emulator software.</li> <li>On-line resources such as YouTube, Khan Academy, Desmos, EdPuzzle, Kahoot, etc.</li> </ul>	

ESTABLISHED GOALS	Tr	ransfer	
CCSS.Math.Content.HSA.SSE.B	Students will be able to independently use their learning to Make sense of a problem by initiating a plan and persevere in solving		
exponents to transform expressions for exponential functions.	Model with mathematics by using the appropriate method Reason abstractly Justify reasoning or understanding by explaining techniques to solving Attend to precision		
CCSS.Math.Content.HSF.IF.C.7. e Graph exponential and			
logarithmic functions, showing	М	eaning	
intercepts and end behavior, and	UNDERSTANDINGS	ESSENTIAL QUESTIONS	
period, midline, and amplitude.	Students will understand that	Students will keep considering	
CCSS.Math.Content.HSF.IF.C.8. b Use the properties of exponents to interpret expressions for exponential functions.	<ul> <li>Exponential equations can be solved by getting a common base or by using logarithms.</li> <li>Logarithms are used to represent exponents, which could not be solved.</li> <li>Properties of logarithms relate to the properties of exponents.</li> </ul>	<ul> <li>What is the value of an exponential equation in the real-world?</li> <li>How are exponents and logarithms related?</li> <li>How does the relationship between exponential and logarithmic functions help us?</li> <li>What are some real world applications of</li> </ul>	
CCSS.Math.Content.HSF.BF.B.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and	<ul> <li>Interest on banking accounts is modeled with exponential functions as well as archaeology, oceanography, and manufacturing applications to name a few.</li> </ul>	What are some real-word applications of logarithmic and exponential functions?	
exponents.	Acq	quisition	
	<ul><li>Students will know</li><li>The process of solving exponential</li></ul>	<ul> <li>Students will be skilled at</li> <li>Changing expressions to have the same</li> </ul>	
	equations by getting a common base and	base in order to solve exponential	

<ul> <li>by using logarithms.</li> <li>A logarithm is a way to represent exponents.</li> <li>The properties of logarithms.</li> </ul>	<ul> <li>equations.</li> <li>Changing expressions from exponential form to logarithmic form and vice-versa.</li> <li>Evaluating logarithmic expressions.</li> <li>Applying the properties of logarithms to solve exponential equations.</li> </ul>
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Code	Evaluative Criteria	Assessment Evidence
	Further information:	PERFORMANCE TASK(S):
		Students will show that they really understand evidence of
I, M, A	Scoring Rubric used to evaluate successful understanding of the process and criteria for a desired outcome.	<u>Goal</u> : Students will apply concepts of exponential equations in order to calculate continuous versus yearly compounded interest.
		<ul> <li><u>Role</u>: Students will take on the role of a financial analyst</li> <li><u>Audience</u>: Business Manager</li> <li><u>Situation</u>: Students are given different scenarios that they will compare the two different types of interest rates.</li> <li><u>Product</u>: Analysis of the different scenarios.</li> <li><u>Standard for Success</u>: rubric based on understanding, accuracy, communication of results, presentation of evidence to support claim.</li> <li><u>To Differentiate</u>: Provide different problems with different levels of difficulty from which students can choose.</li> </ul>

М	Thorough understanding of identifying values	OTHER EVIDENCE:
	that are restricted from the domain,	Students will show they have achieved Stage 1 goals by
	simplifying a rational expression, types of	
	polynomials.	<ul> <li>Alternative assessment projects such as "find the mistakes", explain the process, posters, and real world applications</li> </ul>
T, M, A	Thorough understanding of steps and processes to simplify, multiply, divide, add, subtract, and solve equations.	<ul> <li>Review of standardized test questions to prep students for the challenge of the SAT and ACT exams</li> <li>Quizzes</li> </ul>
		UNIT Test
T, M, A	Accurate application of content/process to arrive at the correct mathematical solution.	<ul><li> "Do Now" questions/opening</li><li> Activities</li></ul>
		Questioning
T, M, A	Selection of evidence that is relevant to content and standardized test processes.	<ul> <li>Self-assessment</li> <li>Smartboard activities, (Kahoot, Quizlet, etc.)</li> </ul>
	1	

Code	Pre-Assessment		
М	<ul> <li>Teacher checks for prerequisite and prior knowledge via warm-ups and entrance tickets on simplifying ration exponents, evaluating expressions and using linear models.</li> <li>Questioning activities, such as basic problems with simplifying expressions with exponents.</li> <li>As the lessons progress, students can also be given questions such as "Find the mistakes"</li> <li>Warm-ups and skill checks contain review of previous material during the unit to ensure retention and mastery, and check on vertical alignment with prior curriculum.</li> </ul>		
	Summary of Key Learning Events and Instruction Student success at transfer meaning and acquisition depends on	Progress Monitoring	
М	<ul> <li>Teacher uses independent/guided practice via supplemental worksheets to review simplifying expressions with exponents.</li> </ul>	<ul> <li>Monitoring class work through board work, group work, questioning, warm-ups, and walk-arounds.</li> <li>Homework check to assess common errors to</li> </ul>	
T,A	• Students work independently and in teacher created groups to complete practice problems that review exponents. Students will use think-pair-share to compare and discuss their answers	<ul> <li>inform future instruction.</li> <li>Check prerequisite knowledge throughout the unit using warm-up problems and questioning activities.</li> </ul>	
М	<ul> <li>Teacher walks around and monitors student progress,</li> </ul>	Differentiate through purposeful or flexible	

T.A	<ul> <li>assists individual students, and models examples when needed for the class.</li> <li>Students will individually complete problems on solving</li> </ul>	
- )	exponential equations. Students will volunteer their	
	solutions and will explain the process they used.	•
М	<ul> <li>Teacher gives warm-up questions on exponents as a way to introduce exponential equations.</li> </ul>	•
Μ	<ul> <li>Teacher models different examples of exponential equations that have the same base and the steps to solving them.</li> </ul>	
M,T	<ul> <li>Teacher has the class graph the equation y = 2x and its inverse as a way of introducing the graph of an exponential equation and a logarithm.</li> </ul>	
Μ	<ul> <li>Teacher models how to solve and evaluate logarithmic equations and expressions by changing to exponential form and by applying the properties of logarithms</li> </ul>	
T, A	<ul> <li>Students will practice evaluating and solving logarithmic expressions and equations by various activities such as independent activities such as independent practice, board work, think-pair-share, and/or use of white boards.</li> </ul>	
	<u>Resources:</u> All Resources and materials must adhere to all New Milford Board of Education policies and regulations and are subject to New Milford Board of Education approval. Resources and materials must be researched and vetted by the writers and department heads prior to submission for approval.	
	<ul> <li>Textbook: Blitzer, Robert. Precalculus Second Edition, Upper Saddle River, NJ: Pearson, 2004.</li> <li>Supplemental activities from the textbook resources</li> <li>Teacher-made supplemental activities on applications, performance tasks, and chapter review</li> <li>Graphing calculator TI Emulator software.</li> <li>On-line resources such as YouTube, Khan Academy, Desmos, EdPuzzle, Kahoot, etc.</li> </ul>	

- Check for understanding: board and whiteboard activities, or reflections and exit tickets.
- Strategic Questioning: Ask students higher-order questions such as "how" and "why," so the teacher can discern the level and extent of the students' understanding.

ESTABLISHED GOALS	Transfer		
CCSS.MATH.CONTENT.HSF.TF. A.1 Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle. CCSS.MATH.CONTENT.HSF.TF. A.2 Explain how the unit circle in the coordinate plane enables the	Students will be able to independently use their learning to Make sense of a problem by initiating a plan and persevere in solving Model with mathematics by using the appropriate method Reason abstractly Justify reasoning or understanding by explaining techniques to solving Attend to precision		
extension of trigonometric	M	eaning	
extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. CCSS.MATH.CONTENT.HSF.TF A.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$ , $\pi/4$ and $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for $\lambda$	<ul> <li>UNDERSTANDINGS Students will understand that</li> <li>Right triangles, which are similar, will have the same relationships between specific pairs of sides (i.e., opposite: hypotenuse).</li> <li>Right triangle trigonometry has many uses and applications in the real-world.</li> <li>Right triangle trigonometry has many uses and applications.</li> <li>The Unit Circle can represent angles of any measure, in degrees or radians, and</li> </ul>	<ul> <li>ESSENTIAL QUESTIONS Students will keep considering</li> <li>Why do the trigonometric ratios hold for all right triangles with the same angle measures?</li> <li>How can the use of right triangles, trigonometric functions, and the Pythagorean Theorem be used to solve real-world problems?</li> <li>How can the use of right triangles, trigonometric functions, and the Pythagorean Theorem be used to solve real</li> </ul>	
$\pi$ + <i>x</i> , and $2\pi$ - <i>x</i> in terms of their values for <i>x</i> , where <i>x</i> is any real number CCSS.MATH.CONTENT.HSG.S RT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied	<ul> <li>is cyclic.</li> <li>Trigonometric functions of specific angles relate to specific points and values on the unit circle.</li> <li>Graphs of the trigonometric functions are cyclic with certain traits.</li> <li>Sine and Cosine graphs produce "waves."</li> </ul>	<ul> <li>world problems?</li> <li>What is the unit circle and why is it important in trigonometry?</li> <li>Why do the graphs of trigonometric functions look the way they do?</li> <li>How do the graphs of sine and cosine apply to real life applications?</li> <li>What are the uses of inverse trigonometric functions?</li> </ul>	

Acquisition	
Students will know	Students will be skilled at
<ul> <li>Pythagorean Theorem</li> <li>Definitions of the six trigonometric functions</li> <li>Radian measures</li> <li>Angles, points, and trigonometric values on the unit circle</li> <li>Reference angles</li> <li>Methods to graphing sine, cosine, tangent, cosecant, and secant, and their general behaviors</li> <li>Inverse trigonometric functions and the restrictions on their ranges</li> <li>Compositions of trigonometric functions</li> </ul>	<ul> <li>Using the Pythagorean Theorem and right triangle trigonometry to solve right triangles</li> <li>Defining the six trigonometric functions</li> <li>Constructing a unit circle and identify angles in both degree and radian measures</li> <li>Converting degrees to radians (and vice versa)</li> <li>Identifying specific points on the unit circle</li> <li>Defining the trigonometric functions as related to the x and y coordinates and radius on the unit circle</li> <li>Using reference angles and definitions of the trigonometric functions to find the specific values on the unit circle.</li> <li>Filing in the trigonometric table for values of special and quadrantal angles</li> <li>Graphing sine, cosine, tangent, cosecant, and secant functions and identify special characteristics such as amplitude, period, phase shift, and vertical shift</li> <li>Identifying the domain and range of inverse trigonometric functions</li> <li>Finding the exact values of inverse trigonometric functions</li> <li>Using graphing calculators to check graphs of trigonometric values and to find approximate solutions to problems</li> <li>Solving real-world applications involving right triangle trigonometry.</li> </ul>

Code	Evaluative Criteria	Assessment Evidence
	Further information:	PERFORMANCE TASK(S):
		Students will show that they really understand evidence of
T, M, A	Scoring Rubric used to evaluate successful understanding of the process and criteria for a desired outcome.	Task 1: <u>Goal</u> : To use right triangle trigonometry to solve real-world application problems <u>Role</u> : Students will take on the role of a surveyor <u>Audience</u> : Land development company <u>Situation</u> : Given various situations, you are to calculate unknown distances to report to the land development company for construction purposes. <u>Product</u> : Calculated distances with work shown. <u>Standard for Success</u> : rubric based on understanding, accuracy, communication of results, presentation of evidence to support claim. <u>To Differentiate</u> : Provide different problems with different levels of difficulty from which students can choose. <u>Task 2:</u> <u>Goal</u> : To calculate your personal biorhythm chart for the current month <u>Role</u> : Students will take on the role of a social scientist <u>Audience</u> : School staff <u>Situation</u> : You are to convince school staff whether or not your personal academic performance will be stronger or weaker based on your biorhythm <u>Product</u> : Your completed biorhythm <u>Standards for Success</u> : rubric based on understanding, accuracy, communication of results, presentation of evidence to support claim.

		OTHER EVIDENCE: Students will show they have achieved Stage 1 goals by
Μ	Thorough understanding of identifying values that are restricted from the domain, simplifying a rational expression, types of polynomials.	<ul> <li>Alternative assessment projects such as "find the mistakes", explain the process, posters, and real world applications</li> <li>Review of standardized test questions to prep students for the challenge of the SAT and ACT exams</li> <li>Quizzes</li> </ul>
T, M, A	Thorough understanding of steps and processes to simplify, multiply, divide, add, subtract, and solve equations.	<ul> <li>UNIT Test</li> <li>"Do Now" questions/opening</li> <li>Activities</li> <li>Questioning</li> </ul>
T, M, A	Accurate application of content/process to arrive at the correct mathematical solution.	<ul> <li>Self-assessment</li> <li>Smartboard activities, (Kahoot, Quizlet, etc.)</li> </ul>
T, M, A	Selection of evidence that is relevant to content and standardized test processes.	

Code	Pre-Assessment
М	<ul> <li>Teacher checks for prerequisite and prior knowledge via warm-ups and entrance tickets</li> <li>Questioning activities, such as basic problems with right triangle trigonometry and simplifying radicals.</li> <li>As the lessons progress, students can also be given questions such as "Find the mistakes"</li> <li>Warm-ups and skill checks contain review of previous material during the unit to ensure retention and mastery, and check on vertical alignment with prior curriculum.</li> </ul>

	Summary of Key Learning Events and Instruction	Progress Monitoring
	Student success at transfer meaning and acquisition depends on	
		<ul> <li>Monitoring class work through board work,</li> </ul>
Μ	<ul> <li>Teacher reviews the Pythagorean Theorem and right</li> </ul>	group work, questioning, warm-ups, and
	triangle trigonometry.	walk-arounds.
M, A	<ul> <li>Students will practice solving right triangles using</li> </ul>	<ul> <li>Homework check to assess common errors to</li> </ul>
	trigonometry by working in teacher created groups.	inform future instruction.
T, M, A	<ul> <li>Teacher gives review and practice problems as class work</li> </ul>	<ul> <li>Check prerequisite knowledge throughout the</li> </ul>
	to find missing sides and angles. Lesson leads into the	unit using warm-up problems and questioning
	introduction of the three reciprocal trigonometric functions,	activities.
	and applications of trigonometry will be discussed.	<ul> <li>Differentiate through purposeful or flexible</li> </ul>
T, M, A	<ul> <li>Students will also identify the values of the reciprocal</li> </ul>	grouping, use of diagrams and explanations to
	functions and will use their calculators to find specific	demonstrate understanding and active lessons
	values.	involving discovery, scaffolding, jigsaw
M, A	<ul> <li>Teacher introduces the concept of the Unit Circle by first</li> </ul>	activities and use of hands-on manipulatives
	discussing radian measure as a representation of the	<ul> <li>Check for understanding: board and</li> </ul>
	length of the arc on the circle.	whiteboard activities, or reflections and exit
M, A	<ul> <li>Teacher leads the class through the discovery of the</li> </ul>	tickets.
	relationship between degrees and radians and how to	Strategic Questioning: Ask students
	convert degrees to radians and radians to degrees.	higher-order questions such as "how" and
M, A	<ul> <li>Students will work independently on changing measures</li> </ul>	"why," so the teacher can discern the level and
	Tool degrees to radians and radians to degrees.	extent of the students' understanding
M, A	<ul> <li>Teacher continues to demonstrate the relations on the Unit Circle to points on the circle and angle measures</li> </ul>	
	• After a review of special right triangles, the teacher	
I, IVI, A	<ul> <li>After a review of special right thangles, the teacher models how to find specific points given particular</li> </ul>	
	reference angles	
тма	<ul> <li>Students will complete the Unit Circle chart with specific</li> </ul>	
I, IVI, A	degree measure radian measure and the coordinate of	
	the associated points. Students may collaborate with a	
тм	partner on their solutions	
1, 101	<ul> <li>Teacher prepares materials (garland, laminated color</li> </ul>	
	coded cards with degree measures, radian measures, and	
	coordinates of points) for the Unit Circle activity where	
Т. М	students physically construct a model of the Unit Circle.	
,	• Students will work cooperatively as a group to construct	
	the Unit Circle in the rotunda using garland and laminated	

T, M, A	values on the circle.	
	<ul> <li>Students will then use the Unit Circle and reference</li> </ul>	
	angles to fill in the trigonometric table. Students will then	
	play the "Move It" game where they must move to a	
M, A	specific value on the circle.	
	<ul> <li>Teacher reviews the relationships of trigonometric</li> </ul>	
	functions in right triangles and then shows the connection	
	with the x, y, and r values of the Unit Circle. Lesson leads	
	into applications of the trigonometric functions to any point	
	in the coordinate plane, which the teacher models and	
Т, М, А	explains.	
	<ul> <li>Students will apply the definitions of the trigonometric</li> </ul>	
N4 0	functions to the Unit Circle. Students will then find the	
IVI, A	values of the trigonometric functions at any value.	
	• With the help of the graphing calculator, the teacher reads	
	Discussion on the general shape of the curves, their	
	periodic behavior, and their amplitude, period, phase shift	
МА	and vertical shift occurs	
ivi, 7 (	<ul> <li>Teacher leads the class through examples on how to</li> </ul>	
T, M, A	graph sine and cosine functions.	
	<ul> <li>Students will work at the board to practice graphing sine</li> </ul>	
	and cosine functions, identifying the amplitude, period,	
M, A	phase shift, and vertical shift.	
	<ul> <li>Teacher has the class make a t-table to graph the tangent</li> </ul>	
	curve. The general shape of the curve and its period will	
M, A	be discussed.	
	<ul> <li>Teacher models how to graph the secant and cosecant</li> </ul>	
	functions by using the sine and cosine graphs as	
М, А	neipers."	
	<ul> <li>Students will graph y=tanx as well as various cosecant and accept outvoe by working in teacher created groups</li> </ul>	
IVI, A	<ul> <li>Teacher models how to use the graphing calculator and</li> </ul>	
	<ul> <li>reacher models now to use the graphing calculator and trigonometric table to find approximate and exact values</li> </ul>	
	of inverse trigonometric functions. Teacher also explains	
т. м. а	how to find composite trigonometric values.	
-,,	<ul> <li>Students will work in pairs to find inverse and composite</li> </ul>	

M, A	<ul> <li>trigonometric functions using their calculators and the trigonometric table.</li> <li>Teacher determines cooperative groups for various activities during this unit.</li> </ul>	
	Resources: All Resources and materials must adhere to all New Milford Board of Education policies and regulations and are subject to New Milford Board of Education approval. Resources and materials must be researched and vetted by the writers and department heads prior to submission for approval.	
	<ul> <li>Textbook: Blitzer, Robert. Precalculus Second Edition, Upper Saddle River, NJ: Pearson, 2004.</li> <li>Supplemental activities from the textbook resources</li> <li>Teacher-made supplemental activities on applications, performance tasks, and chapter review</li> <li>Graphing calculator TI Emulator software.</li> <li>On-line resources such as YouTube, Khan Academy, Desmos, EdPuzzle, Kahoot, etc.</li> </ul>	

ESTABLISHED GOALS	Transfer	
CC 9-12 G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	Students will be able to independently use their learning Make sense of a problem by initiating a Model with mathematics by using the ap Reason abstractly Justify reasoning or understanding by ex Attend to precision	plan and persevere in solving propriate method xplaining techniques to solving
CC 9-12 N-Q.3 Choose a level of		
accuracy appropriate to	Meaning	
when reporting quantities.	UNDERSTANDINGS Students will understand that	ESSENTIAL QUESTIONS Students will keep considering
	<ul> <li>The Law of Sines and Law of Cosines apply to non-right triangles and can be used to find missing lengths or angles.</li> </ul>	<ul> <li>How can the use of trigonometric functions be extended to solve word problems and triangles with no right angles?</li> </ul>
	Acquisition	
	<ul> <li>Students will know</li> <li>Law of Sines</li> <li>Law of Cosines</li> <li>Applications of the Law of Sines and Law of Cosines</li> </ul>	<ul> <li>Students will be skilled at</li> <li>Solving a triangle for missing sides or angles using the Law of Sines and the Law of Cosines</li> <li>Applying the ambiguous of the Law of Sines to determine if there are none, one, or two possible triangles</li> </ul>

Code	Evaluative Criteria	Assessment Evidence	
	Further information:	PERFORMANCE TASK(S):	
		Students will show that they really understand evidence of	
T, M, A	Scoring Rubric used to evaluate successful understanding of the process and criteria for a desired outcome.	<ul> <li><u>Goal</u>: To find unknown values in specific real-world situations <u>Role</u>: Students will take on the role of a surveyor <u>Audience</u>: Land development company <u>Situation</u>: Given various situations, use the Laws of Sines and Cosines to calculate values that are otherwise non-measurable (e.g., calculate the distance between two landmarks that have a lake between them).</li> <li><u>Product</u>: Calculated distances with work shown.</li> <li><u>Standard for Success</u>: rubric based on understanding, accuracy, communication of results, presentation of evidence to support claim.</li> <li><u>To Differentiate</u>: Provide different problems with different levels of difficulty from which students can choose.</li> </ul>	
		Students will show they have achieved Stage 1 goals by	
М	Thorough understanding of identifying values that are restricted from the domain, simplifying a rational expression, types of polynomials.	<ul> <li>Alternative assessment projects such as "find the mistakes", explain the process, posters, and real world applications</li> <li>Review of standardized test questions to prep students for the challenge of the SAT and ACT exams</li> </ul>	
T, M, A	Thorough understanding of steps and processes to simplify, multiply, divide, add, subtract, and solve equations.	<ul> <li>Quizzes</li> <li>UNIT Test</li> <li>"Do Now" questions/opening</li> <li>Activities</li> </ul>	
T, M, A	Accurate application of content/process to arrive at the correct mathematical solution.	<ul> <li>Questioning</li> <li>Self-assessment</li> <li>Smartboard activities, (Kahoot, Quizlet, etc.)</li> </ul>	

T, M, A	Selection of evidence that is relevant to
	content and standardized test processes.

Code	Pre-Assessment	
Μ	<ul> <li>Teacher checks for prerequisite and prior knowledge via warm-ups and entrance tickets</li> <li>Questioning activities, such as basic problems with trigonometry.</li> <li>As the lessons progress, students can also be given questions such as "Find the mistakes"</li> <li>Warm-ups and skill checks contain review of previous material during the unit to ensure retention and mastery, and check on vertical alignment with prior curriculum.</li> </ul>	
	Summary of Key Learning Events and Instruction	Progress Monitoring
M, A	<ul> <li>Teacher introduces the Law of Sines by having students solve a right triangle. Discussion takes place about solving a non-right triangle, and the teacher gives the</li> </ul>	<ul> <li>Monitoring class work through board work, group work, questioning, warm-ups, and walk-arounds.</li> <li>Homework check to assess common errors to</li> </ul>
M, A	<ul> <li>Teacher leads the class through discovery of the relationship between angles and sides in using the Law of Sines in different cases (AAS, ASA, SSA).</li> </ul>	<ul> <li>Inform future instruction.</li> <li>Check prerequisite knowledge throughout the unit using warm-up problems and questioning activities.</li> </ul>
M, A	<ul> <li>Students will work as a whole group to practice some examples on the Law of Sines (non ambiguous case)</li> </ul>	Differentiate through purposeful or flexible     grouping use of diagrams and explanations to
M, A	<ul> <li>Teacher reviews the ambiguous case for the Law of Sines and demonstrates why there are possibly no triangles, one triangle, or two triangles.</li> </ul>	demonstrate understanding and active lessons involving discovery, scaffolding, jigsaw activities and use of hands-on manipulatives
M, A	<ul> <li>Teacher models examples of using the Law of Sines equation with no triangle and with two triangles.</li> </ul>	<ul> <li>Check for understanding: board and whiteboard activities or reflections and exit</li> </ul>
T, M, A	<ul> <li>Students will practice examples of the ambiguous case of the Law of Sines by working in small teacher created groups.</li> </ul>	<ul> <li>Strategic Questioning: Ask students higher-order questions such as "how" and</li> </ul>
M, A	<ul> <li>Teacher leads the class through discovery of the relationship between angles and sides in using the Law of Cosines and discusses when to use it (SSS, SAS cases).</li> </ul>	"why," so the teacher can discern the level and extent of the students' understanding.

M, A T, M, A T, M, A	<ul> <li>Teacher models an example where a side should be found first, and then one where an angle should be found first.</li> <li>Students will solve triangles using the Law of Cosines and will compare their answers with a partner.</li> <li>Students will work independently to complete the performance task related to applications with the Law of Sines and Law of Cosines.</li> </ul>	
	<u>Resources:</u> All Resources and materials must adhere to all New Milford Board of Education policies and regulations and are subject to New Milford Board of Education approval. Resources and materials must be researched and vetted by the writers and department heads prior to submission for approval.	
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