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## 2016 Mississippi <br> College- and CareerReadiness Standards for Mathematics

Effective Date: 2016-2017 School Year


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# 2016 Mississippi College- and Career-Readiness Standards for Mathematics 

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# Introduction 

Mission Statement

The Mississippi Department of Education is dedicated to student success including the improvement of student achievement in mathematics in order to produce citizens who are capable of making complex decisions, solving complex problems, and communicating fluently in a technological society. The 2016 Mississippi College- and Career-Readiness Standards for Mathematics ("The Standards") provide a consistent, clear understanding of what students are expected to know and be able to do by the end of each grade level and course. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that students need for success in college and careers and to compete in the global economy.

## Purpose

In an effort to closely align instruction for students who are progressing toward postsecondary study and the workforce, the 2016 Mississippi College- and Career-Readiness Standards for Mathematics include grade- and course-specific standards for K-12 mathematics.

The primary purpose of this document is to provide a basis for curriculum development for Grades K-12 mathematics teachers, outlining what students should know and be able to do by the end of each grade level and course. Courses for grades K-12 are based on the Mississippi College- and Career-Readiness Standards (MS CCRS) for Mathematics. Mississippi-specific courses that were revised to align with the (MS CCRS) for Mathematics include Pre-Calculus (renamed Algebra III) and Calculus.

The Southern Regional Education Board (SREB) Math Ready course is included as a transition to college mathematics courses.

The content of this document is centered on the mathematics domains of Counting and Cardinality (Grade K), Operations and Algebraic Thinking; Numbers and Operations in Base Ten (Grades K-5); Numbers and Operations-Fractions (Grades 3-5); Measurement and Data (Grades K-5); Ratios and Proportional Relationships (Grades 6-7); the Number System, Expressions \& Equations, Geometry, Statistics \& Probability (Grades 6-8); Functions (Grade 8), and the high school conceptual categories of Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics \& Probability. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

The required year for the 2016 Mississippi College- and Career-Readiness Standards for Mathematics is school year 2016-2017.

## Technology

The Mississippi Department of Education (MDE) strongly encourages the use of technology in all mathematics classrooms. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

The appropriate use of instructional technology is integrated throughout the 2016 Mississippi College- and Career-Readiness Standards for Mathematics. Teaching strategies at each grade level and in every secondary course incorporate technology in the form of calculators, software, or on-line internet resources. The graphing calculator is an integral part of mathematics courses beginning with Grade 6.

The MDE believes strongly in the Principles and Standards for School Mathematics Technology Principle of the National Council for Teachers of Mathematics (NCTM):
"Calculators and computers are reshaping the mathematical landscape, and school mathematics should reflect those changes. Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalization or abstraction. In the mathematics classrooms envisioned in Principles and Standards, every student has access to technology to facilitate his or her mathematics learning. Technology also offers options for students with special needs. Some students may benefit from the more constrained and engaging task situations possible with computers. Students with physical challenges can become much more engaged in mathematics using special technologies. Technology cannot replace the mathematics teacher, nor can it be used as a replacement for basic understandings and intuitions. The teacher must make prudent decisions about when and how to use technology and should ensure that the technology is enhancing students' mathematical thinking."
(NCTM, 2013, http://www.nctm.org.)

## Acknowledgements

COMMITTEE MEMBERS (2015)
The Mississippi Department of Education gratefully acknowledges the following individuals who provided feedback in developing the 2016 Mississippi College- and Career-Readiness Standards for Mathematics.

Christy Bennett Jennifer Bennett
Suzanne Breazeale
Stephanie Brewer
Shelita Brown, Ph.D.
Libby Cook
Kelli Crain
Marla Davis, Ph.D.
Donna Diaz
Elizabeth Fulmer
Cheryl Heavener
Shauna Hedgepeth
Sidney Margaret Holbert, Ph.D.
Leslie Holloway
Jessica Ivy, Ph.D.
Suzanne Jennings
Carol Ladner
Patricia Lampkin
Joycelyn Lorenzo
Sissy Lynn
Meredith "Paige" Phillips
Amie Powell
Kim Price
Chetonia Pryor
Deia Sanders
Kristen Schrimpshire
Alice Steimle, Ph.D.
Sharon Sutherlin
Jennifer Wilson

DeSoto County School District<br>Calhoun County School District Neshoba County<br>Petal Public School District<br>Mississippi Department of Education<br>Hinds County School District<br>Mississippi Department of Education<br>Mississippi Department of Education<br>Biloxi School District<br>Mississippi Department of Education<br>South Tippah School District<br>Lamar County School District<br>Mississippi College<br>Pearl School District<br>Mississippi State University<br>Pascagoula-Gautier School District<br>Mississippi Department of Education<br>Humphreys County School District<br>Gulfport School District<br>Madison County School District<br>Ocean Springs School District<br>Marion County School District<br>Clinton School District<br>Coahoma AHS School District<br>Simpson County School District<br>Quitman School District<br>University of Mississippi<br>Bay-Waveland School District<br>Rankin County School District

## Acknowledgements

COMMITTEE MEMBERS (2013)
The Mississippi Department of Education gratefully acknowledges the following individuals who provided feedback in developing the 2015 Mississippi College- and Career-Readiness Standards for Mathematics.

Lisa Amacker<br>Richard Baliko<br>Stephanie Brewer<br>Angela Cooley<br>Tammi Crosetti<br>Marla Davis, Ph.D.<br>Melinda Gann, Ph.D.<br>Roy Gill<br>Trecina Green<br>David Jay Herbert<br>Susan Lee, Ed.D.<br>JoAnn Malone<br>Jean Massey<br>Nathan Oakley<br>Kerri Pippin<br>Jenny Simmons<br>Alice Steimle, Ph.D.<br>LaVerne Ulmer, Ph.D.<br>Jennifer Wilson

Petal Public School District<br>Mississippi Department of Education<br>Petal Public School District<br>Jones County Junior College<br>Rankin County School District<br>Mississippi Department of Education<br>Mississippi State University<br>Harrison County Public Schools<br>Mississippi Department of Education Delta State University<br>Institutions of Higher Learning Mississippi Department of Education<br>Mississippi Department of Education<br>Mississippi Department of Education<br>Jones County Junior College<br>Lee County School District<br>University of Mississippi<br>Jones County Junior College<br>Rankin County School District



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## College- and CareerReadiness Standards for Mathematics Overview

## College- and Career-Readiness Standards for Mathematics

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is "a mile wide and an inch deep." These Standards are a substantial answer to that challenge. Aiming for clarity and specificity, these Standards endeavor to follow a design that not only stresses conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the laws of arithmetic to structure those ideas.

## Understanding Mathematics

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a+b)(x+y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a+b+c)(x+y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific expectations but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary for college and/or careers. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with reading disabilities the use of Braille, screen reader technology, or other assistive devices should be made available. In addition, while writing, these students should have access to a scribe, computer, or speech-to-text technology in their classroom. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of College- and Career-Readiness for all students.

## Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## 2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize-to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents-and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argumentexplain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## 4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a
website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 6. Attend to precision

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well-remembered
$7 \times 5+7 \times 3$, in preparation for learning about the distributive property. In the expression $x^{2}+$ $9 x+14$, older students can see the 14 as $2 \times 7$ and the 9 as $2+7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5-3$ (x$y)^{2}$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.
8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1,2)$ with slope 3 , middle school students might abstract the equation $(y-2) /(x-1)=3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1),(x-1)\left(x^{2}+x+1\right)$, and $(x-1)\left(x^{3}+x^{2}+x+1\right)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the 2016 Mississippi Collegeand Career-Readiness Standards for Mathematics

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to the Standards in mathematics instruction.

The Standards are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

## Modeling (High School Courses only)

Modeling standards are noted throughout the high school courses with an asterisk (*). Modeling links classroom mathematics and statistics to everyday life, work, and decisionmaking. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.


Making mathematical models is a Standard for Mathematical Practice, and specific Modeling standards appear throughout the high school standards. The basic modeling cycle above involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.


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# College- and CareerReadiness Standards for Mathematics Scaffolding Document 

# 2016 Mississippi College- and Career-Readiness Standards Scaffolding Document 

Purpose

The primary purpose of the 2016 Mississippi College- and Career-Readiness Standards Scaffolding Document is to provide teachers with a deeper understanding of the Standards as they plan for classroom instruction. Based on the 2016 Mississippi College- and CareerReadiness Standards for Mathematics, this document provides a close analysis of the requirements for student mastery. Because of the rigor and depth of the Standards, scaffolding instruction to meet the needs of all learners is essential to individual success. The Scaffolding Document will aid teachers' understanding of how to teach the Standards through a natural progression of student mastery. The Scaffolding Document is located at http://www.mde.k12.ms.us/ESE/ccr.

Organization of the 2016 Mississippi College- and Career-Standards Scaffolding Document
The 2016 Mississippi College- and Career-Readiness Standards Scaffolding Document is divided by grade level. Within each grade level, the Scaffolding Document is separated into the mathematical domains Counting and Cardinality (Grade K), Operations and Algebraic Thinking (Grades K-5); Numbers and Operations in Base Ten (Grades K-5); Numbers and Operations-Fractions (Grades 3-5); Measurement and Data (Grades K-5); Ratios and Proportional Relationships (Grades 6-7); the Number System, Expressions \& Equations, Geometry, Statistics \& Probability (Grades 6-8); Functions (Grade 8), and the high school conceptual categories of Number and Quantity, Algebra, Functions, Geometry, and Statistics \& Probability.

Each standard is then broken down into three categories: Prerequisite Knowledge, Conceptual Understanding, and Evidence of Knowledge. The Prerequisite Knowledge column lists the skills that students should have mastered in previous grades in order to work towards mastery of the grade-specific standard. In other words, this column details what a student needs to KNOW before mastering the grade-specific standard. The Conceptual Understanding column explains the deeper understanding of concepts-not actions or skills-that are required for mastery of the grade specific standard. In other words, this column explains what a student needs to UNDERSTAND before mastering the grade-specific standard. The last column, Evidence of Knowledge, explains what student mastery looks like, including what work a student produces to exhibit mastery of the grade-specific standard. In other words, this column describes what a student needs to $D O$ to show mastery of the grade-specific standard.

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# College- and CareerReadiness Standards for Mathematics (Grades K-5) 

## Fluency/Fluently Defined

Throughout the 2016 Mississippi College- and Career-Readiness Standards for Mathematics Grades K-5 standards, the words fluency and fluently will appear in bold, italicized, and underlined font (for example: $\underline{f l u e n t l y}$ ). With respect to student performance and effective inclass instruction, the expectations for mathematical fluency are explained below:

Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend one or more grades earlier in the standards than the grade when fluency is finally expected.

Wherever the word fluently appears in a MS CCR content standard, the word means quickly and accurately. It is important to understand that this is not explicitly tied to assessment purposes, but means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow: Fluent isn't halting, stumbling, or reversing oneself.

A key aspect of fluency in this sense is that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all students to meet the standards that call explicitly for fluency.

## Grade 4

In Grade 4, instruction should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. Each critical area is described below.
(1) Students generalize their understanding of place value to $1,000,000$, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, and area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.
(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., $15 / 9=5 / 3$ ), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

## Grade 4 (continued)

The content of this document is centered on the mathematics domains of Counting and Cardinality (Grade K), Operations and Algebraic Thinking; Numbers and Operations in Base Ten (Grades K-5); Numbers and Operations-Fractions (Grades 3-5);
Measurement and Data (Grades K-5); Ratios and Proportional Relationships (Grades 67); the Number System, Expressions \& Equations, Geometry, Statistics \& Probability (Grades 6-8); Functions (Grade 8), and the high school conceptual categories of Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics \&
Probability. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

## Grade 4

| Operations and Algebraic Thinking (OA) |  |
| :---: | :---: |
| Use the four operations with whole numbers to solve problems |  |
| 4.OA. 1 | Interpret a multiplication equation as a comparison, e.g., interpret $35=5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5 . Represent verbal statements of multiplicative comparisons as multiplication equations. |
| 4.OA. 2 | Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. ${ }^{1}$ |
| 4.OA. 3 | Solve multistep (two or more operational steps) word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. |
| Gain familiarity with factors and multiples |  |
| 4.OA. 4 | Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range $1-100$ is a multiple of a given one-digit number. Determine whether a given whole number in the range $\quad 1-100$ is prime or composite. |
| Generate and analyze patterns |  |
| 4.OA. 5 | Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. |
| Number and Operations in Base Ten ${ }^{2}$ (NBT) |  |
| Generalize place value understanding for multi-digit whole numbers |  |
| 4.NBT. 1 | Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70=10$ by applying concepts of place value and division. |
| 4.NBT. 2 | Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. |
| 4.NBT. 3 | Use place value understanding to round multi-digit whole numbers to any place. |

## Grade 4

| Use place value understanding and properties of operations to perform multi-digit arithmetic |  |
| :---: | :---: |
| 4.NBT. 4 | Fluently add and subtract (including subtracting across zeros) multi-digit whole numbers using the standard algorithm. |
| 4.NBT. 5 | Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. |
| 4.NBT. 6 | Find whole-number quotients and remainders with up to four-digit dividends and onedigit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. |
| Number and Operations-Fractions ${ }^{3}$ (NF) |  |
| Extend understanding of fraction equivalence and ordering |  |
| 4.NF. 1 | Recognizing that the value of " $n$ " cannot be 0 , explain why a fraction a/b is equivalent to a fraction $(\mathrm{n} \times \mathrm{a}) /(\mathrm{n} \times \mathrm{b})$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. |
| 4.NF. 2 | Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1 / 2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. |
| Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers |  |
| 4.NF. 3 | Understand a fraction $a / b$ with $a>1$ as a sum of fractions $1 / b$. <br> a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. <br> b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model (including, but not limited to: concrete models, illustrations, tape diagram, number line, area model, etc.). Examples: $3 / 8=1 / 8+1 / 8+1 / 8 ; 3 / 8=1 / 8+2 / 8 ; 21 / 8=1+1+1 / 8=8 / 8+8 / 8+$ 1/8. <br> c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. <br> d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. |

## Grade 4

| 4.NF. 4 | Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. <br> a. Understand a fraction $a / b$ as a multiple of $1 / b$. For example, use a visual fraction model to represent $5 / 4$ as the product $5 \times(1 / 4)$, recording the conclusion by the equation $\quad 5 / 4=5 \times(1 / 4)$. <br> b. Understand a multiple of $a / b$ as a multiple of $1 / b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times(2 / 5)$ as $6 \times(1 / 5)$, recognizing this product as $6 / 5$. (In general, $n \times$ $(a / b)=(n \times a) / b$. <br> c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $3 / 8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers do you expect your answer to lie? |
| :---: | :---: |
| Understand decimal notation for fractions, and compare decimal fractions |  |
| 4.NF. 5 | Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and $100 .{ }^{4}$ For example, express $3 / 10$ as $30 / 100$, and add $3 / 10+4 / 100=34 / 100$. |
| 4.NF. 6 | Use decimal notation for fractions with denominators 10 or 100 . For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. |
| 4.NF. 7 | Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. |
| Measurement and Data (MD) |  |
| Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit |  |
| 4.MD. 1 | Know relative sizes of measurement units within one system of units including km, m, $\mathrm{cm}, \mathrm{mm} ; \mathrm{kg}, \mathrm{g}, \mathrm{mg}$; lb, oz.; I, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in . Express the length of a 4 ft snake as 48 in . Generate a conversion table for feet and inches listing the number pairs $(1,12),(2,24),(3,36)$,... |
| 4.MD. 2 | Use the four operations to solve word problems involving <br> - intervals of time <br> - money <br> - distances <br> - liquid volumes <br> - masses of objects <br> including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. |

## Grade 4

| 4.MD. 3 | Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. |
| :---: | :---: |
| Represent and interpret data |  |
| 4.MD. 4 | Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, $1 / 8)$. Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection. |
| Geometric measurement: understand concepts of angle and measure angles |  |
| 4.MD. 5 | Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: <br> a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. <br> b. An angle that turns through $n$ one-degree angles is said to have an angle measure of $n$ degrees. |
| 4.MD. 6 | Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. |
| 4.MD. 7 | Recognize angle measure as additive. When an angle is decomposed into nonoverlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. Example: Find the missing angle using an equation. |
| Geometry (G) |  |
| Draw and identify lines and angles, and classify shapes by properties of their lines and angles |  |
| 4.G. 1 | Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. |

## Grade 4

| 4.G.2 | Classify two-dimensional figures based on the presence or absence of parallel or <br> perpendicular lines, or the presence or absence of angles of a specified size. Recognize <br> right triangles as a category, and identify right triangles. |
| :--- | :--- |
| 4.G.3 | Recognize a line of symmetry for a two-dimensional figure as a line across the figure <br> such that the figure can be folded along the line into matching parts. Identify line- <br> symmetric figures and draw lines of symmetry. |

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## Additional Resource

## 2016 Mississippi College- and Career-Standards Scaffolding Document

The primary purpose of the 2016 Mississippi College- and Career-Readiness Standards Scaffolding Document is to provide teachers with a deeper understanding of the Standards as they plan for classroom instruction. Based on the 2016 Mississippi College- and CareerReadiness Standards for Mathematics, this document provides a close analysis of the requirements for student mastery. Because of the rigor and depth of the Standards, scaffolding instruction to meet the needs of all learners is essential to individual success. The Scaffolding Document will aid teachers' understanding of how to teach the Standards through a natural progression of student mastery. The Scaffolding Document can be found at http://www.mde.k12.ms.us/ESE/ccr.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

[^0]:    ${ }^{1}$ See Glossary, Table 2.
    ${ }^{2}$ Grade 4 expectations in this domain are limited to whole numbers less than or equal I to $1,000,000$.
    ${ }^{3}$ Grade 4 expectations in this domain are limited to fractions with denominators $2,3,4,5,6,8,10,12$, and 100 .
    ${ }^{4}$ Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

