1.1 Place Value, Rounding, Comparing Whole Numbers

Place Value

Example: The number 13,652,103 would look like

Millions			Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
	1	3	6	5	2	1	0	3

We'd read this in groups of three digits, so this number would be written
thirteen million six hundred fifty two thousand one hundred and three

Example: What is the place value of 4 in 6,342,105?
The 4 is in the ten-thousands place

Example: Write the value of two million, five hundred thousand, thirty six
2,500,036

Rounding

When we round to a place value, we are looking for the closest number that has zeros in the digits to the right.

Example: Round 173 to the nearest ten.
Since we are rounding to the nearest ten, we want a 0 in the ones place. The two possible values are 170 or 180. 173 is closer to 170, so we round to 170.

Example: Round 97,870 to the nearest thousand.
The nearest values are 97,000 and 98,000. The closer value is 98,000.

Example: Round 5,950 to the nearest hundred.
The nearest values are 5,900 or 6,000. 5,950 is exactly halfway between, so by convention we round up, to 6,000.

Comparing

To compare to values, we look at which has the largest value in the highest place value.

Example: Which is larger: 126 or 132?
Both numbers are the same in the hundreds place, so we look in the tens place. 132 has 3 tens, while 126 only has 2 tens, so 132 is larger. We write $126 < 132$, or $132 > 126$.

Example: Which is larger: 54 or 236?
Here, 54 includes no hundreds, while 236 contains two hundreds, so 236 is larger.
$54 < 236$, or $236 > 54$

Worksheet – 1.1 Place value, rounding, comparing   Name: ________________________________

1) Write out in words: 13,904

2) Write out in words: 30,000,000

3) Write out in words: 13,000,000,000

4) Write the number: sixty million, three hundred twelve thousand, two hundred twenty five

5) Round to the nearest ten: 83,974

6) Round to the nearest hundred: 6,873

Round 8,499 to the nearest 7) ten 8) hundred 9) thousand

Determine which number is larger. Write < or > between the numbers to show this.

10) 13 21

11) 91 87

12) 136 512

13) 6,302,542 6,294,752

14) six thousand five hundred twenty three six thousand ninety five

1.2 Adding and Subtracting Whole numbers

Note: If you are happy with the way you've always added or subtracted whole number, by all means continue doing it the same way!

Adding by Grouping

Example: Add 352 + 179

We can break this apart:

Add the hundreds:	300+100 =	400
Add the tens:	50 + 70 =	120
Add the ones:	2 + 9 =	11
Add the resulting pieces	=	531

Adding by Rearranging

The idea that 2 + 3 is the same as 3 + 2 is called the <u>commutative property</u> for addition.

Example: Add 17 + 15 + 23

We can rearrange the order to be 17 + 23 + 15

Since 7+3 = 10, this makes things a bit easier

17 + 23 + 15 = 40 + 15 = 55

Subtracting using Expanded form

Example: Subtract 352 - 169

We can write this as:

```
      300 + 50 + 2
-     100 + 60 + 9
```

We can't take 9 from 2, so we borrow 10 from the 50

```
      300 + 40 + 12
-     100 + 60 + 9
```

Likewise we can't take 60 from 40, so we borrow 100 from the 300

```
      200 + 140 + 12
-     100 +  60  + 9
--------------------------
      100 + 80 + 3 = 183
```

Subtracting by Adjusting Values

Example: Subtract 162 - 138

If we add 2 to both numbers, the difference will be the same, but easier to compute

```
  162 + 2  =  164
– 138 + 2  =  140
------------------------
               24
```

Worksheet – 1.2 Add / Subtract Whole numbers Name: ________________________________

Calculate. Use whatever techniques make sense to you.

1) 513 + 268

2) 1704 + 521

3) 88 + 26 + 32 + 4

4) 12,000 + 312

5) 92 – 75

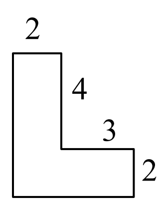
6) 1824 – 908

7) 3000 – 73

8) 903 – 170

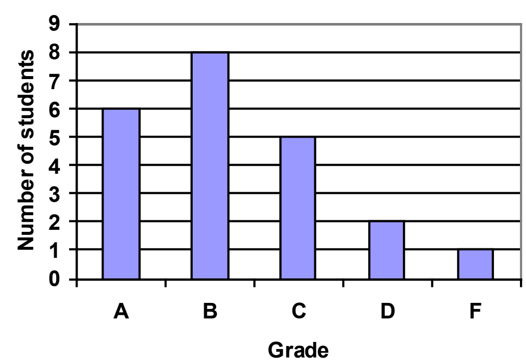
9) 100 – 13 + 17

11) Find the perimeter of the shape shown.



12) This year I used 606 kWh of electricity in August. Last year I used 326 kWh. How much more electricity did I use this year?

13) The bar graph shows grades on a class activity. How many students scored a C or better?



1.3 Multiplication of Whole Numbers

There are three common ways of writing “5 times 3”: 5×3, $5 \cdot 3$, and $(5)(3)$

Multiplying as repeated addition

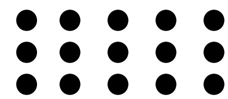
Example: Multiply $42 \cdot 3$

This is equivalent to 42 + 42 + 42 = 126

Multiplying by thinking about area

Example: Multiply $5 \cdot 3$

We can think of this as 3 groups of 5 objects or 5 groups of 3 objects:



This is also why we multiply to find the <u>area</u> of rectangle.

$5 \cdot 3 = 15$

Multiplying using place values

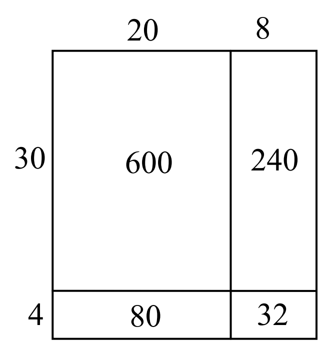
The idea that $5 \cdot (20+3) = 5 \cdot 20 + 5 \cdot 3$ is called the <u>distributive property</u>.

Example: Multiply $28 \cdot 6$

We can write this as $28 \cdot 6 = (20+8) \cdot 6$ Then

$(20+8) \cdot 6 = 20 \cdot 6 + 8 \cdot 6$

$= 120 + 48 = 168$



Example: Multiply $28 \cdot 34$

We can think of this as $20 \cdot 30 + 20 \cdot 4 + 30 \cdot 8 + 8 \cdot 4$

This can be thought of as areas, as pictured to the right

$600 + 80 + 240 + 32 = 952$

You also write this in the more “traditional” way. By working with place values, you can avoid having to carry. It’s more writing, but less likely to

Example: Multiply $162 \cdot 24$

```
 162
x 24
2000   100 x 20
 400   100 x  4
1200    60 x 20
 120    60 x  4
  40     2 x 20
+  8     2 x  4
3768
```

Multiplying large numbers

When you have a lot of trailing zeros, you can multiply without them, then tack on the extra zeros at the end.

Example: Multiply $14000 \cdot 30$

$14 \cdot 3 = 42$, and there were 4 trailing zeros, so the result is: 420,000

Worksheet – 1.3.1 Multiplying Whole numbers Name: ________________________________

Calculate. Use whatever techniques make sense to you.

1) $47 \cdot 6$

2) $152 \cdot 8$

3) $25 \cdot 74$

4) $346 \cdot 27$

5) $300 \cdot 6000$

6) $5200 \cdot 40$

7) Estimate the value of $72{,}132 \cdot 6{,}817$ by rounding the values first

For each problem, decide if you should add or multiply, then calculate the result

8) You and three friends each order a \$3 slice of pie. How much is your total bill?

9) You ordered a burger for \$9 and your friend ordered pasta for \$12. How much is your total bill?

10) You earn \$12 an hour. How much do you make in an 8 hours shift?

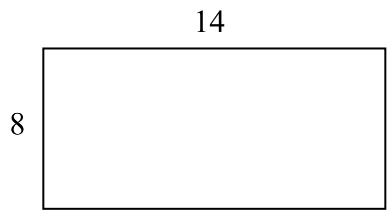
11) You need to order 120 new t-shirts for each of 6 stores. How many t-shirts do you need to order?

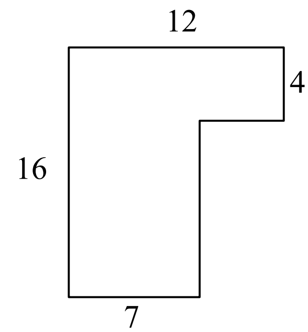
12) The average American uses 920 kilowatt hours of electricity each month. How much do they use in a year?

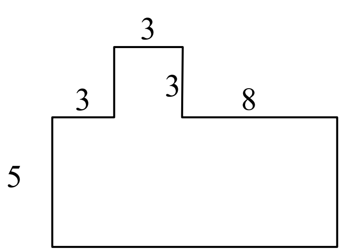
13) You buy two bottles of headache medicine, one with 100 pills, and the other with 75 pills. How many pills do you have total?

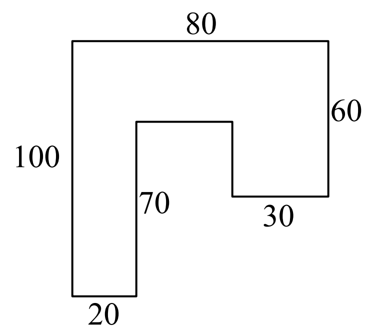
Worksheet – 1.3.2 Areas – Whole numbers Name: ___________________________________

Find the area of each figure.

1) 

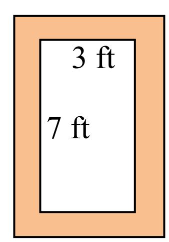
2) 

3) 

4) 

6) A bedroom measures 12 feet by 13 feet. What is the area of the room? If carpet costs $3 per square foot, how much will it cost to carpet this room?

7) You're going to build a 3 foot by 7 foot garden. If you want to surround it with a 1 foot wide brick border, how many square feet of brick will you need?
(see the picture – the shaded part is the brick)



1.4 Dividing Whole Numbers

There are 3 common ways of writing "6 divided by 3": $6 \div 3$, $\frac{6}{3}$, and $3\overline{)6}$

You can think of division as splitting something into equal groups.
Example: You have 12 cookies and 3 kids are going to share them. How many does each kid get?
We divide the 12 cookies into 3 groups: $12 \div 3 = 4$ cookies per kid

Sometimes we might have something left over; this is a remainder
Example: Find $66 \div 7$
Dividing 66 items into 7 piles, we couldn't put 10 in each pile, since that would require $7 \cdot 10 = 70$ items. So we can put 9 items in each pile, using up $7 \cdot 9 = 63$ items, and we have 3 items left over.
$66 \div 7 = 9$ with remainder 3

Long Division

Example: Divide $8\overline{)2896}$

We're trying to find a number that multiplied by 8 will give 2896
Since $1000 \cdot 8 = 8000$, we know the number is smaller than 1000.
What's the biggest 100's number that, when multiplied by 8, is <u>not bigger than</u> 2896?
$200 \cdot 8 = 1600$, $300 \cdot 8 = 2400$, $400 \cdot 8 = 3200$.
Looks like 300 is the biggest hundred, so we write a 3 in the hundreds place, multiply $300 \cdot 8$ and write the result 2400 below the 2896.
Now subtract. This is how much is still left.

```
     3
 8 )2896
   -2400
     496
```

Next, we're going to look for the biggest 10's number that, when multiplied by 8, is <u>not bigger than</u> 496. $50 \cdot 8 = 400$, $60 \cdot 8 = 480$, $70 \cdot 8 = 560$
Looks like 60 is the biggest ten that fits, so we write a 6 in the tens place, multiply $60 \cdot 8 = 480$ and write that below the 496.
Now subtract. This is how much is still left.

```
     36
 8 )2896
   -2400
     496
    -480
      16
```

Lastly, we're going to look for the biggest number that, when multiplied by 8, is <u>not bigger than</u> 16. Since $2 \cdot 8 = 16$, we write a 2 in the ones place, multiply $2 \cdot 8 = 16$ and write that below the 16, and subtract. Since we have nothing left, there is no remainder.

```
     362
 8 )2896
   -2400
     496
    -480
      16
     -16
       0
```

So $2896 \div 8 = 362$

Division involving zero

If we have nothing, and we divide it into any number of piles, each pile will have nothing, so $0 \div 5 = 0$

However, dividing by zero doesn't make sense. For example think about $5 \div 0$. That's asking what number, when multiplied by 0, gives 5. There isn't one! <u>Dividing by zero is undefined</u>.

Worksheet – 1.4 Dividing Whole numbers Name: ______________________________

Calculate.

1) $56 \div 7$

2) $210 \div 3$

3) $\dfrac{800}{10}$

4) $7\overline{)1225}$

5) $12\overline{)168}$

6) $23\overline{)8303}$

For each problem, decide if you should add, subtract, multiply, or divide, then calculate the result

8) Four roommates agree to split the $1500 rent equally. How much will each pay?

9) A team for the Alzheimer's walk has raised $375. How much more do they need to raise to reach their goal of $1000?

10) A car insurance quote is $744 for six months. How much is that a month?

11) Your friend with a flock of chickens wants to give you 65 eggs. How many egg cartons (the kind that holds a dozen eggs) should you take with you to carry the eggs?

12) If you make $2,240 a month, how much do you make each year?

13) If you make $2,240 a month, how much do make each week (roughly – assume 4 weeks in a month)?

1.5 Exponents, Roots, and Order of Operations

Exponents and Roots

If we have repeated multiplication, like $5 \cdot 5 \cdot 5 \cdot 5$ we can write this more simply using **exponents**: 5^4

Example: Write $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ using exponents

Since we are multiplying 3 times itself 5 times, the base is 3, and the exponent is 5: 3^5

Example: Evaluate 6^3

$6^3 = 6 \cdot 6 \cdot 6 = 36 \cdot 6 = 216$

Undoing squaring a number is finding the **square root**, which uses the symbol $\sqrt{}$. It's like asking "what number times itself will give me this value?" So $\sqrt{36} = 6$ since $6^2 = 36$

Example: Find $\sqrt{81}$

$\sqrt{81} = 9$ because $9^2 = 81$

Order of Operations

When we combine multiple operations, we need to agree on an order to follow, so that if two people calculate $2 + 3 \cdot 4$ they will get the same answer. To remember the order, some people use the mnemonic PEMDAS:

P:	**P**arentheses
E:	**E**xponents and roots
MD:	**M**ultiplication and **D**ivision
AS:	**A**ddition and **S**ubtraction

IMPORTANT!! Notice that multiplication and division have the SAME precedence, as do addition and subtraction. When you have multiple operations of the same level, you work left to right.

Example: Simplify $2 + 3 \cdot 4 - 6 + 10 \div 2$

We start with the multiplication and division: $3 \cdot 4 = 12$ and $10 \div 2 = 5$: $2 + 12 - 6 + 5$

Now we add and subtract from left to right: $14 - 6 + 5$

$8 + 5$

13

Example: Simplify $4(5 + 2) - 6\sqrt{9} + 4^2$

We start with the inside of the parentheses: 5+2=7: $4(7) - 6\sqrt{9} + 4^2$

Next we evaluate the exponents and root: $\sqrt{9} = 3$, $4^2 = 16$: $4(7) - 6 \cdot 3 + 16$

Next we do the multiplications: $28 - 18 + 16$

Lastly add and subtract from left to right: $10 + 16$

26

Worksheet – 1.5 Exponents, roots, order of operations Name: ________________________________

Evaluate.

1) 4^3

2) $\sqrt{49}$

3) $18-12\div 3$

4) $10-3+5$

5) $2\cdot 3\cdot 5$

6) $10\div 2\cdot 5$

7) $2\cdot 3^2$

8) $\frac{\sqrt{100}}{2}$

9) $6-2(8-2\cdot 3)$

10) $3(1+3)^2+4$

11) $4\sqrt{25-4^2}$

12) For a rectangle, the formula Perimeter = 2L+2W is often used, where L is the length and W width. Use this formula to find the perimeter of a rectangle 10 feet long and 4 feet wide.

Write out the mathematical expression that would calculate the answer to each question:

13) A family of four goes out to a buffet, and pays \$10 each for food, and \$2 each for drinks. How much do they pay altogether?

14) Don bought a car for \$1200, spent \$300 on repairs, and sold it for \$2300. How much profit did he make?

1.6 Mean, Median, Mode

The **mean** (sometimes called average) of a set of values is $\frac{\text{Sum of data values}}{\text{Number of data values}}$

Example: Marci's exam scores for her last math class were: 79, 86, 82, 93. The mean of these values would be: (79 + 86 + 82 + 93) divided by 4: $\frac{79+86+82+93}{4}=\frac{340}{4}=85$

Example: On three trips to the store, Bill spent $120, $160, and $35. The mean of these values would be $\frac{120+160+35}{3}=\frac{315}{3}=105$

It would be most correct for us to report that "The mean amount Bill spent was $105 per trip," but it is not uncommon to see the more casual word "average" used in place of "mean".

Median

With some types of data, like incomes or home values, a few very large values can make the mean compute to something much larger than is really "typical". For this reason, another measure, called the **median** is used.

To find the median, begin by listing the data in order from smallest to largest. If the number of data values is odd, then the median is the middle data value. If the number of data values is even, there is no one middle value, so we find the mean of the two middle values.

Example: Suppose Katie went out to lunch every day this week, and spent $12, $8, $72, $6, and $10 (the third day she took the whole office out). To find the median, we'd put the data in order first: $6, $8, $10, $12, $72. Since there are 5 pieces of data, an odd number, the median is the middle value: $10.

Example: Find the median of these quiz scores: 5 10 9 8 6 4 8 2 5 8

We start by listing the data in order: 2 4 5 5 6 8 8 8 9 10

Since there are 10 data values, an even number, there is no one middle number, so we find the mean of the two middle numbers, 6 and 8: $\frac{6+8}{2}=\frac{14}{2}=7$. So the median quiz score was 8.

Mode

The **mode** of a set of data is the value that appears the most often. If not value appears more then once, there is no mode. If more than one value occurs the most often, there can be more than one mode. Because of this, mode is most useful when looking at a very large set of data.

Example: The number of touchdown (TD) passes thrown by each of the 31 teams in the National Football League in the 2000 season are shown below.

37 33 33 32 29 28 28 23 22 22 22 21 21 21 20
20 19 19 18 18 18 18 16 15 14 14 14 12 12 9 6

Looking at these values, the value 18 occurs the most often, appearing 4 times in the list, so 18 is the mode.

Worksheet – 1.6 Mean, median, mode

Name: ________________________________

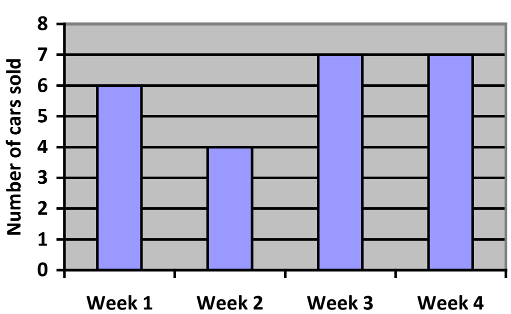
Find the mean, median, and mode of each data set

1) 3, 4, 2, 6, 1, 2

2) 2, 4, 1, 5, 28

3) A small business has five employees, including the owner. Their salaries are $32,000, $40,000, $28,000, $65,000, and $140,000. Find the mean and median salary.

4) The graph shown shows the number of cars sold at a dealership each week this month. Find the mean and median sales per week.

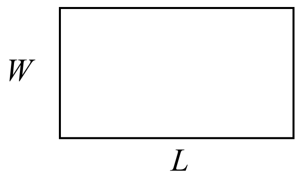


1.7 Areas and Perimeters of Quadrilaterals

Rectangles

Perimeter: $2 \cdot L + 2 \cdot W$

Area: $L \cdot W$



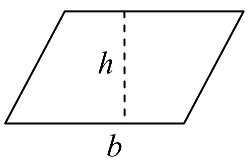
Perimeter: $2 \cdot 4 + 2 \cdot 3 = 8 + 6 = 14$

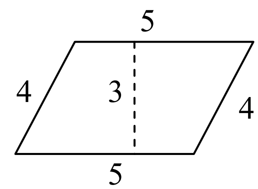
Area: $3 \cdot 4 = 12$

Parallelogram

Perimeter: Sum of the sides

Area: $b \cdot h$





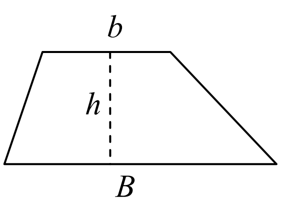
Perimeter: $4 + 5 + 4 + 5 = 18$

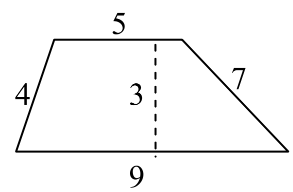
Area: $5 \cdot 3 = 15$

Trapezoid

Perimeter: Sum of the sides

Area: $h\left(\dfrac{b+B}{2}\right)$



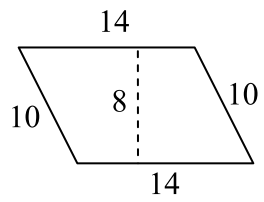


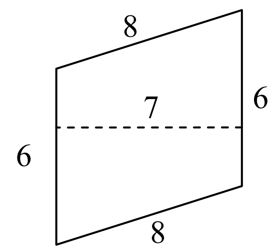
Perimeter: $4 + 5 + 7 + 9 = 25$

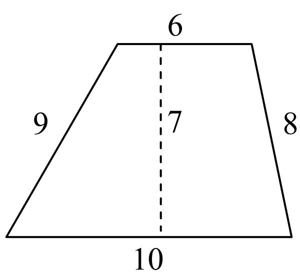
Area: $3\left(\dfrac{5+9}{2}\right) = 3\left(\dfrac{14}{2}\right) = 3(7) = 21$

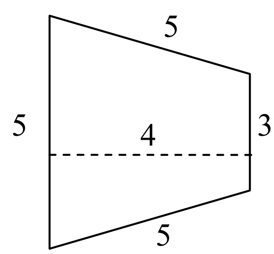
Worksheet – 1.7 Quadrilaterals Name: ______________________________

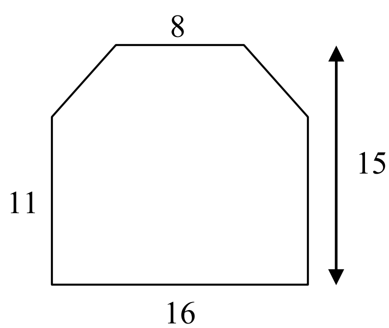
Find the area and perimeter of each figure. Figures may not be drawn accurately to scale.

1) 

2) 

3) 

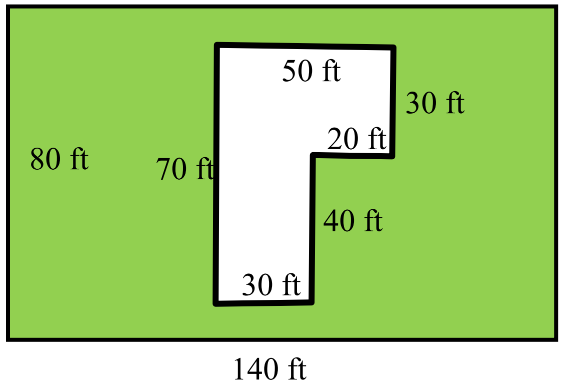
4) 

5) 

Whole numbers activity Names: ________________________________

You are welcome to complete this assignment in groups. You can turn in your answers on a separate sheet of paper or the back of this paper, but make sure I can tell how you came up with your answers, and that I don't have to hunt for your answers.

One of my neighbors is planning a major yard renovation. She wants to re-seed her entire lawn, and fence the two sides and back of her yard. Here an overhead view of her house and yard. Using the picture scale from Google maps, I estimated the dimensions of her yard and house.



For the lawn, she has selected this grass seed. In case you can't read it, one bag covers 1,200 square feet of area, and costs $12 (close enough for our purposes).

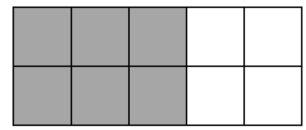
For the fencing, she has selected a nice cedar pre-made panels that are each 8 feet long. Each panel costs $57.

Help my neighbor out – figure out how much grass seed and how many fence panels she needs to buy, and how much the materials are going to cost her.

2.1 Fractions and Mixed Numbers

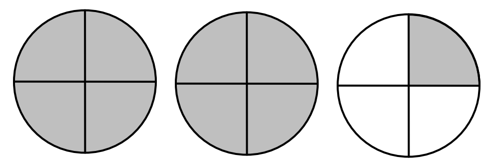
Fractions are a way of representing parts of a whole. For example, if pizza is cut into 8 pieces, and Sami takes 3 pieces, he's taken $\frac{3}{8}$ of the pizza, which we read as "three eighths."

The number on the bottom is called the **denominator**, and indicates how many pieces the whole has been divided into. The number on top is the **numerator**, and shows how many pieces of the whole we have.



Example: What fraction of the large box is shaded?

The box is divided into 10 pieces, of which 6 are shaded, so $\frac{6}{10}$ is shaded.



If we have more than one whole, we often write **mixed numbers**.

Example: In the picture shown, we have two full circles, and a part of a third circle. We commonly write this as $2\frac{1}{4}$, indicating that we have two wholes, and 1 additional quarter.

This mixed number could also be written as an **improper fraction**, which is what we call a fraction where the numerator is equal to or bigger than the denominator. In our circle picture above, we could write the shaded part as $\frac{9}{4}$, indicating that if we divide all the circles into quarters, there are 9 shaded quarters altogether. A **proper fraction** is a fraction where the numerator is smaller than the denominator.

Converting from mixed number to improper fraction

- Multiply the whole number by the denominator of the fraction to determine how many pieces we have in the whole.
- Add this to the numerator of the fraction
- Use this sum as the numerator of the improper fraction. The denominator is the same.

Example: Convert $5\frac{2}{7}$ to an improper fraction.

If we had 5 wholes, each divided into 7 pieces, that'd be $5 \cdot 7 = 35$ pieces.

Adding that to the additional 2 pieces gives 35+2 = 37 total pieces. The fraction would be $\frac{37}{7}$

Converting from improper fraction to mixed number

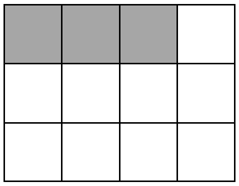
- Divide: numerator ÷ denominator
- The quotient is the whole part of the mixed number
- The remainder is the numerator of the mixed number. The denominator is the same.

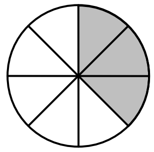
Example: Write $\frac{47}{6}$ as a mixed number. Dividing, 47÷6 = 7 remainder 5. So there are 7 wholes, and 5 remaining pieces, giving the mixed number $7\frac{5}{6}$

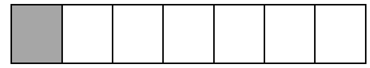
Worksheet – 2.1 Intro to Fractions Name: ______________________________

1) Out of 15 people, four own cats. Write the fraction of the people who own cats.

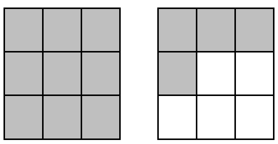
For each picture, write the fraction of the whole that is shaded

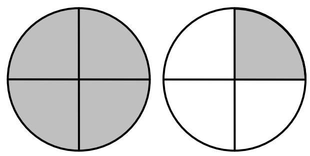
2) 

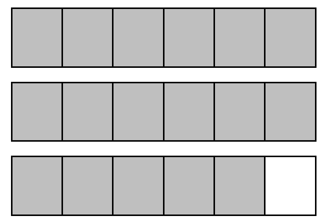
3) 

4) 

For each picture, write the shaded portion as a mixed number <u>and</u> as an improper fraction

5) 

6) 

7) 

Convert each mixed number to an improper fraction

8) $4\frac{3}{4}$

9) $1\frac{7}{16}$

10) $15\frac{1}{2}$

11) $23\frac{5}{8}$

Convert each improper fraction to a mixed number or whole number

12) $\frac{35}{2}$

13) $\frac{15}{6}$

14) $\frac{23}{10}$

15) $\frac{164}{4}$

Measure the length of each bar using a ruler.

16)

17)

18)

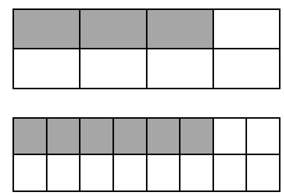
2.2 Simplifying Fractions

To simplify fractions, we first will need to be able to find the **factors** of a number. The factors of a number are all the numbers that divide into it evenly.
Example: Find the factors of 18.
The factors of 18 are 1, 2, 3, 6, 9, 18, since each of those numbers divides into 18 evenly.

When we **factor a number**, we write it is a product of two or more factors.
Example: Factor 24
There are several possibilities: $2\cdot12$, $3\cdot8$, $4\cdot6$, $2\cdot2\cdot2\cdot3$

The last of the factorizations above is called the **prime factorization** because it is written as the product of **prime numbers** – numbers that can't be broken into smaller factors.

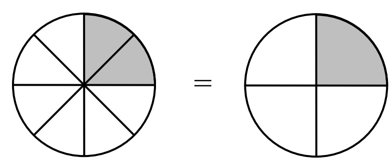
Equivalent fractions
To find equivalent to fractions, we can break our fraction into more or fewer pieces.
For example, by subdividing the rectangle to the right, we see $\frac{3}{8}=\frac{6}{16}$. By doubling the number of total pieces, we double the number of shaded pieces as well.



To find equivalent fractions, multiply or divide **both** the numerator and denominator by the same number.
Example: Write two fractions equivalent to $\frac{2}{8}$

By multiplying the top and bottom by 3, $\frac{2}{8}=\frac{2\cdot3}{8\cdot3}=\frac{6}{24}$

By dividing the top and bottom by 2, $\frac{2}{8}=\frac{2\div2}{8\div2}=\frac{1}{4}$



Example: Write $\frac{3}{5}$ with a denominator of 15

To get a denominator of 15, we'd have to multiply 5 by 3. $\frac{3}{5}=\frac{3\cdot3}{5\cdot3}=\frac{9}{15}$

To simplify fractions to lowest terms, we look for the biggest factor the numerator and denominator have in common, and divide both by that.
Example: Simplify $\frac{12}{18}$

12 and 18 have a common factor of 6, so we divide by 6: $\frac{12}{18}=\frac{12\div6}{18\div6}=\frac{2}{3}$

Alternatively, you can write $\frac{12}{18}=\frac{2\cdot6}{3\cdot6}$ and since $\frac{6}{6}=1$, $\frac{12}{18}=\frac{2\cdot6}{3\cdot6}=\frac{2}{3}$

Example: Simplify $\frac{24}{132}$

If you're not sure of the largest factor, do it in stages: $\frac{24}{132}=\frac{24\div2}{132\div2}=\frac{12}{66}=\frac{12\div6}{66\div6}=\frac{2}{11}$

Worksheet – 2.2 Simplifying Fractions Name: ________________________________

Write all the factors of each number

1) 36 2) 32 3) 120

Find the biggest common factor of each pair of numbers

4) 12 and 8 5) 4 and 12 6) 10 and 25 7) 36 and 27

8) Rewrite $\frac{3}{7}$ with a denominator of 28

9) Rewrite $\frac{18}{24}$ with a denominator of 6

Simplify to lowest terms

10) $\frac{3}{6}$ 11) $\frac{10}{12}$ 12) $\frac{130}{150}$ 13) $\frac{18}{24}$

14) $\frac{40}{56}$ 15) $\frac{28}{49}$ 16) $\frac{27}{54}$ 17) $\frac{70}{126}$

Rewrite each pair of fractions to have the same denominator

18) $\frac{1}{3}$ and $\frac{1}{4}$ 19) $\frac{5}{6}$ and $\frac{3}{8}$ 20) $\frac{3}{20}$ and $\frac{7}{32}$ 21) $\frac{5}{56}$ and $\frac{3}{42}$

2.3 Multiplying Fractions

To multiply two fractions, you multiply the numerators, and multiply the denominators: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

Example: Multiply and simplify $\frac{2}{3} \cdot \frac{5}{8}$

$\frac{2}{3} \cdot \frac{5}{8} = \frac{2 \cdot 5}{3 \cdot 8} = \frac{10}{24}$, which we can simplify to $\frac{5}{12}$

Alternatively, we could have noticed that in $\frac{2 \cdot 5}{3 \cdot 8}$, the 2 and 8 have a common factor of 2, so we can divide the numerator and denominator by 2, often called "cancelling" the common factor: $\frac{2 \cdot 5}{3 \cdot 8} = \frac{1 \cdot 5}{3 \cdot 4} = \frac{5}{12}$

Example: Multiply and simplify $\frac{7}{8} \cdot 6$

It can help to write the whole number as a fraction: $\frac{7}{8} \cdot \frac{6}{1} = \frac{7 \cdot 6}{8 \cdot 1}$. Since 6 and 8 have a factor of 2 in common, we can cancel that factor, leaving $\frac{7 \cdot 3}{4 \cdot 1} = \frac{21}{4}$. This could also be written as the mixed number $5\frac{1}{4}$.

To multiply with mixed numbers, it is easiest to first convert the mixed numbers to improper fractions.

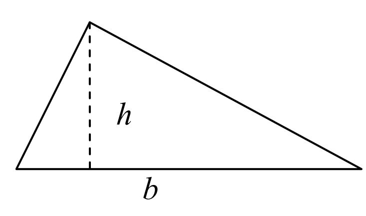
Example: Multiply and simplify $3\frac{1}{3} \cdot 4\frac{4}{5}$

Converting these to improper fractions first, $3\frac{1}{3} = \frac{10}{3}$ and $4\frac{4}{5} = \frac{24}{5}$, so $3\frac{1}{3} \cdot 4\frac{4}{5} = \frac{10}{3} \cdot \frac{24}{5}$

$\frac{10}{3} \cdot \frac{24}{5} = \frac{10 \cdot 24}{3 \cdot 5}$. Since 5 and 10 have a common factor of 5, we can cancel that factor: $\frac{2 \cdot 24}{3 \cdot 1}$.

Since 3 and 24 have a common factor of 3, we can cancel that factor: $\frac{2 \cdot 8}{1 \cdot 1} = \frac{16}{1} = 16$

Areas of Triangles

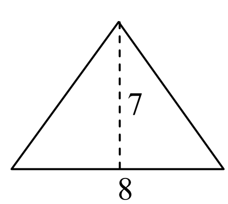


To find the area of a triangle, we can use the formula $Area = \frac{1}{2} \cdot b \cdot h$

Example: Find the area of the triangle shown

The area would be $\frac{1}{2} \cdot 8 \cdot 7$

$\frac{1}{2} \cdot \frac{8}{1} \cdot \frac{7}{1} = \frac{56}{2} = 28$



Worksheet – 2.3 Multiplying Fractions Name: ________________________________

Multiply and simplify

1) $\frac{2}{5} \cdot \frac{3}{4}$

2) $\frac{5}{6} \cdot \frac{1}{3}$

3) $\frac{5}{12} \cdot \frac{9}{10}$

4) $\frac{3}{10} \cdot \frac{2}{5} \cdot \frac{5}{9}$

5) $12 \cdot \frac{2}{3}$

6) $10 \cdot \frac{4}{15}$

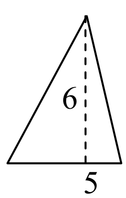
7) $3\frac{1}{2} \cdot \frac{4}{5}$

8) $8\frac{1}{6} \cdot 4\frac{2}{7}$

9) $4 \cdot 2\frac{3}{5}$

Find the area of each shape

10)



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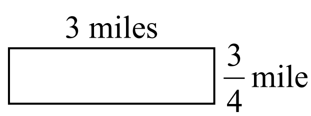
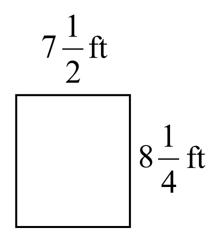


12)



13) Legislature can override the governor’s veto with a 2/3 vote. If there are 49 senators, how many must be in favor to override a veto?

14) A recipe calls for 2½ cups flour, ¾ cup of sugar, and 2 eggs. How much of each ingredient do you need to make half the recipe?

2.4 Dividing Fractions

To find the **reciprocal** of a fraction, we swap the numerator and denominator

Example: Find the reciprocal of $\frac{5}{12}$, $\frac{1}{4}$, and 5

The reciprocal of $\frac{5}{12}$ is $\frac{12}{5}$. The reciprocal of $\frac{1}{4}$ is $\frac{4}{1}=4$. The reciprocal of $5=\frac{5}{1}$ is $\frac{1}{5}$.

To find the **reciprocal** of a mixed number, first write it as an improper fraction

Example: Find the reciprocal of $3\frac{1}{4}$

$3\frac{1}{4}=\frac{13}{4}$, so the reciprocal is $\frac{4}{13}$

To divide two fractions, you find the reciprocal of the number you're dividing by, and multiply the first number times that reciprocal of the second number.

Example: Divide and simplify $\frac{5}{8}\div\frac{5}{6}$

We find the reciprocal of $\frac{5}{6}$ and change this into a multiplication problem: $\frac{5}{8}\cdot\frac{6}{5}=\frac{5\cdot 6}{8\cdot 5}=\frac{1\cdot 3}{4\cdot 1}=\frac{3}{4}$

Example: Divide and simplify $\frac{3}{4}\div\frac{1}{8}$

We find the reciprocal of $\frac{1}{8}$ and change this into a multiplication problem: $\frac{3}{4}\cdot\frac{8}{1}=\frac{3\cdot 8}{4\cdot 1}=\frac{3\cdot 2}{1\cdot 1}=\frac{6}{1}=6$

Example: Divide and simplify $5\frac{1}{2}\div 1\frac{1}{3}$

Rewriting the mixed numbers first as improper fractions, $\frac{11}{2}\div\frac{4}{3}$

We find the reciprocal of $\frac{4}{3}$ and change this into a multiplication problem: $\frac{11}{2}\cdot\frac{4}{3}=\frac{11\cdot 4}{2\cdot 3}=\frac{11\cdot 2}{1\cdot 3}=\frac{22}{3}=7\frac{1}{3}$

Example: You have 5 cups of flour, and a batch of cookies requires $1\frac{3}{4}$ cups of flour. How many batches can you make?

We need to divide: $5\div 1\frac{3}{4}$. Rewriting, $5\div\frac{7}{4}=5\cdot\frac{4}{7}=\frac{5}{1}\cdot\frac{4}{7}=\frac{20}{7}=2\frac{6}{7}$. You can make 2 batches of cookies. You almost have enough for 3 batches, so you might be able to get away with 3.

Example: Making a pillow requires ¾ yard of fabric. How many pillows can you make with 12 yards of fabric?

We need to divide: $12\div\frac{3}{4}$. Rewriting, $12\div\frac{3}{4}=12\cdot\frac{4}{3}=\frac{12}{1}\cdot\frac{4}{3}=\frac{4}{1}\cdot\frac{4}{1}=\frac{16}{1}=16$.

You can make 16 pillows with 12 yards of fabric.

Worksheet – 2.4 Dividing Fractions Name: ________________________________

Divide and simplify

1) $\frac{3}{5} \div \frac{1}{4}$

2) $\frac{7}{8} \div \frac{7}{12}$

3) $\frac{9}{10} \div \frac{3}{5}$

4) $18 \div \frac{2}{3}$

5) $\frac{7}{8} \div 14$

6) $3\frac{1}{4} \div \frac{1}{6}$

7) $2\frac{2}{5} \div 4\frac{1}{3}$

8) $8\frac{1}{2} \div 6$

Decide if each question requires multiplication or division and then answer the question

13) One dose of eyedrops is $\frac{1}{8}$ ounce. How many ounces are required for 40 doses?

14) One dose of eyedrops is $\frac{1}{8}$ ounce. How many doses can be administered from 4 ounces?

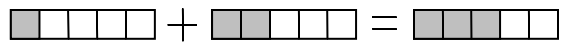
15) A building project calls for 1½ foot boards. How many can be cut from a 12 foot long board?

16) A cupcake recipe yielding 24 cupcakes requires $2\frac{3}{4}$ flour. How much flour will you need if you want to make 30 cupcakes? (this may be a two-step question)

2.5 Add / Subtract Fractions with Like Denominator

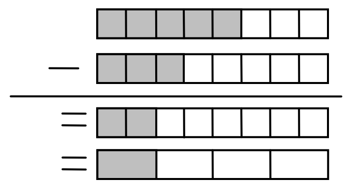
We can only add or subtract fractions with **like** denominators. To do this, we add or subtract the number of pieces of the whole. The denominator remains the same: $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ and $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$

Example: Add and simplify $\frac{1}{5}+\frac{2}{5}$



$\frac{1}{5}+\frac{2}{5}=\frac{1+2}{5}=\frac{3}{5}$

Example: Subtract and simplify $\frac{5}{8}-\frac{3}{8}$



$\frac{5}{8}-\frac{3}{8}=\frac{5-3}{8}=\frac{2}{8}=\frac{1}{4}$

To add mixed numbers, add the whole parts and add the fractional parts. If the sum of the fractional parts is greater than 1, combine it with the whole part

Example: Add and simplify $3\frac{7}{9}+2\frac{5}{9}$

Adding the whole parts $3+2=5$. Adding the fractional parts, $\frac{7}{9}+\frac{5}{9}=\frac{7+5}{9}=\frac{12}{9}=1\frac{3}{9}=1\frac{1}{3}$.

Now we combine these: $5+1\frac{1}{3}=6\frac{1}{3}$

To subtract mixed numbers, subtract the whole parts and subtract the fractional parts. You may need to borrow a whole to subtract the fractions

Example: Subtract and simplify $8\frac{4}{5}-3\frac{3}{5}$

Since $\frac{4}{5}$ is larger than $\frac{3}{5}$, we don't need to borrow. $8-3=5$, and $\frac{4}{5}-\frac{3}{5}=\frac{1}{5}$, so $8\frac{4}{5}-3\frac{3}{5}=5\frac{1}{5}$

Example: Subtract and simplify $5\frac{1}{4}-3\frac{3}{4}$

Since $\frac{1}{4}$ is smaller than $\frac{3}{4}$, we need to borrow. We can say $5\frac{1}{4}=4+1\frac{1}{4}=4\frac{5}{4}$. Now we can subtract:

$4-3=1$ and $\frac{5}{4}-\frac{3}{4}=\frac{2}{4}=\frac{1}{2}$, so $5\frac{1}{4}-3\frac{3}{4}=1\frac{1}{2}$

<u>Alternatively</u>, you can add or subtract mixed numbers my converting to improper fractions first:

$5\frac{1}{4}-3\frac{3}{4}=\frac{21}{4}-\frac{15}{4}=\frac{6}{4}=\frac{3}{2}=1\frac{1}{2}$

Worksheet – 2.5 Add/Subt Fractions Like Denom Name: ________________________________

Add or Subtract and simplify

1) $\frac{2}{5}+\frac{1}{5}$

2) $\frac{3}{10}+\frac{5}{10}$

3) $\frac{6}{7}+\frac{4}{7}$

4) $\frac{7}{8}+\frac{5}{8}$

5) $4\frac{1}{8}+3\frac{5}{8}$

6) $1\frac{3}{5}+7\frac{4}{5}$

7) $2\frac{1}{6}+3\frac{5}{6}$

8) $2\frac{2}{9}+3$

9) $\frac{7}{9}-\frac{2}{9}$

10) $\frac{7}{8}-\frac{5}{8}$

11) $5\frac{5}{6}-3\frac{1}{6}$

12) $4\frac{1}{4}-2\frac{3}{4}$

13) $1\frac{1}{3}-\frac{2}{3}$

14) $4\frac{2}{5}-2$

15) $6-3\frac{1}{4}$

2.6 Part 1 Least Common Multiple

To compare or add fractions with different denominators, we first need to give them a **common denominator**. To prevent numbers from getting really huge, we usually like to find the **least common denominator**. To do this, we look for the **least common multiple**: the smallest number that is a multiple of both denominators.

Method 1: Lucky guess / intuition
In this approach, perhaps you look at the two numbers and you immediately know the smallest number that both denominators will divide into.

Example: Give $\frac{1}{6}$ and $\frac{3}{10}$ a common denominator.

Perhaps by looking at this, you can immediately see that 30 is the smallest multiple of both numbers; the smallest number both will divide evenly into. To give $\frac{1}{6}$ a denominator of 30 we multiply by $\frac{5}{5}$: $\frac{1}{6} \cdot \frac{5}{5} = \frac{5}{30}$.

To give $\frac{3}{10}$ a denominator of 30, we multiply by $\frac{3}{3}$: $\frac{3}{10} \cdot \frac{3}{3} = \frac{9}{30}$

Method 2: List the multiples
In this approach, we list the multiples of a number (the number times 2, times 3, times 4, etc.) and look for the smallest value that shows up in both lists.

Example: Give $\frac{1}{12}$ and $\frac{5}{18}$ a common denominator.

Listing the multiples of each:
12: 12 24 <u>36</u> 48 60 72 96
18: 18 <u>36</u> 54 72 90 108

While they have both 36 and 72 as common multiples, 36 is the least common multiple. To give 12 a denominator of 36 we multiply top and bottom by 3; to give 18 a denominator of 36 we multiply top and bottom by 2. $\frac{1}{12} \cdot \frac{3}{3} = \frac{3}{36}$, $\frac{5}{18} \cdot \frac{2}{2} = \frac{10}{36}$

Method 3: List prime factors
We list the prime factors of each number, then use each prime factor the greatest number of times it shows up in either factorization to find the least common multiple.

Example: Find the least common multiple of 40 and 36.

Breaking each down,
$40 = 4 \cdot 10 = 2 \cdot 2 \cdot 2 \cdot 5$
$36 = 4 \cdot 9 = 2 \cdot 2 \cdot 3 \cdot 3$

Our least common multiple will need three factors of 2, two factors of 3, and one factor of 5:
$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 360$

Method 4: Common factors
In the above approach, after noticing 40 and 36 had a factor of 4 in common, we might have noticed that 9 and 10 had no other common factors, so the least common multiple would be $4 \cdot 9 \cdot 10 = 360$. We only use common factors once in the least common multiple.

Worksheet – 2.6p1 Least Common Multiples Name: ________________________________

Find the least common multiple of each pair of numbers

1) 3 and 7

2) 4 and 10

3) 12 and 16

4) 20 and 30

5) 9 and 15

6) 15 and 18

Give each pair of fractions a common denominator

7) $\frac{5}{7}$ $\frac{9}{14}$

8) $\frac{3}{4}$ $\frac{5}{8}$

9) $\frac{3}{8}$ $\frac{1}{6}$

10) $\frac{1}{10}$ $\frac{4}{15}$

11) $\frac{11}{18}$ $\frac{3}{16}$

12) $\frac{5}{72}$ $\frac{7}{60}$

2.6 Part 2 Add / Subtract Fractions with Unlike Denominator

Since can only add or subtract fractions with **like** denominators, if we need to add or subtract fractions with unlike denominators, we first need to give them a common denominator.

Example: Add and simplify $\frac{1}{4}+\frac{1}{2}$

Since these don't have the same denominator, we identify the least common multiple of the two denominators, 4, and give both fractions that denominator. Then we add and simplify. $\frac{1}{4}+\frac{1}{2}=\frac{1}{4}+\frac{2}{4}=\frac{1+2}{4}=\frac{3}{4}$

Example: Subtract and simplify $\frac{5}{8}-\frac{7}{12}$

The least common multiple of 8 and 12 is 24. We give both fractions this denominator and subtract.

$\frac{5}{8}-\frac{7}{12}=\frac{15}{24}-\frac{14}{24}=\frac{1}{24}$

Example: Add and simplify $\frac{3}{4}+\frac{7}{12}$

We give these a common denominator of 12 and add: $\frac{3}{4}+\frac{7}{12}=\frac{9}{12}+\frac{7}{12}=\frac{9+7}{12}=\frac{16}{12}$

This can be reduced and written as a mixed number: $\frac{16}{12}=\frac{4}{3}=1\frac{1}{3}$

To add and subtract mixed numbers with unlike denominators, give the fractional parts like denominators, then proceed as we did before.

Example: Add and simplify $2\frac{2}{3}+5\frac{3}{4}$

Rewriting the fractional parts with a common denominator of 12: $2\frac{8}{12}+5\frac{9}{12}$

Adding the whole parts $2+5=7$. Adding the fractional parts, $\frac{8}{12}+\frac{9}{12}=\frac{17}{12}=1\frac{5}{12}$.

Now we combine these: $7+1\frac{5}{12}=8\frac{5}{12}$

Example: Subtract and simplify $6\frac{1}{3}-4\frac{5}{6}$

Rewriting the fractional parts with a common denominator of 6: $6\frac{4}{12}-4\frac{10}{12}$

Since $\frac{10}{12}$ is smaller than $\frac{4}{12}$, we borrow: $6\frac{4}{12}=5+1\frac{4}{12}=5+\frac{16}{12}$

$5-4=1$, and $\frac{16}{12}-\frac{10}{12}=\frac{6}{12}=\frac{1}{2}$, so $6\frac{4}{12}-4\frac{10}{12}=1\frac{1}{2}$

Worksheet – 2.6p2 Add/Subt Fractions Unlike Denom Name: ________________________________

Add or Subtract and simplify

1) $\frac{2}{5}+\frac{1}{3}$

2) $\frac{1}{2}+\frac{1}{6}$

3) $\frac{3}{8}+\frac{1}{6}$

4) $\frac{9}{14}+\frac{20}{21}$

5) $3\frac{1}{4}+2\frac{1}{2}$

6) $8\frac{2}{3}+6\frac{3}{4}$

7) $\frac{4}{5}-\frac{7}{10}$

8) $\frac{5}{6}-\frac{2}{9}$

9) $3\frac{7}{12}-1\frac{1}{4}$

10) $4\frac{1}{3}-2\frac{5}{7}$

11) $4-\frac{5}{6}$

12) $6-3\frac{1}{4}$

Worksheet – Fractions Order of Ops Name: ________________________________

Simplify

1) $\left(\frac{2}{3}\right)^2$

2) $\frac{1}{5}+\frac{3}{4}\cdot\frac{2}{5}$

3) $\frac{1}{3}(5+2)$

4) $4\left(\frac{7}{8}-\frac{1}{4}\right)$

5) $\frac{3}{5}\cdot\left(\frac{1}{6}+\frac{1}{4}\right)$

6) $5\cdot\left(\frac{1}{2}\right)^2$

7) $\dfrac{\frac{2}{3}+\frac{4}{5}}{3}$

8) $\dfrac{\frac{5}{6}}{\frac{1}{5}+\frac{3}{10}}$

9) $4\cdot\left(7+2\frac{1}{2}\right)$

10) A room measures 20½ feet long, and 15¾ feet wide. Find the area <u>and</u> perimeter

11) Jean's three pea plants measure 6½, 5¼, and 4 inches tall. Find the mean (average) height

Fractions Activity Names: ________________________

(adapted from MITE material)

You are having a get together and are expecting 30 guests. You plan on serving Banana Bread, Chocolate Chip Cookies, and Sugar Cookies. Using the three recipes given, work with your group to create recipe cards to feed 30 people. Next, total up the ingredients needed. Then, check to see how much of each product needs to be purchased based on what is already on hand.

Banana Bread	Chocolate Chip Cookies	Sugar Cookies
3 bananas	$2\frac{1}{2}$ cups flour	$1\frac{1}{3}$ cup sugar
$\frac{1}{3}$ cup melted butter	1 teaspoon baking soda	$\frac{1}{2}$ cup butter
$\frac{2}{3}$ cup sugar	$\frac{3}{4}$ teaspoon salt	$\frac{1}{4}$ teaspoon baking soda
1 egg	$\frac{3}{4}$ cup sugar	1 egg
$\frac{3}{4}$ teaspoon vanilla	1 cup butter	$\frac{1}{2}$ teaspoon vanilla
$\frac{1}{2}$ teaspoon baking soda	1 teaspoon vanilla	$1\frac{1}{2}$ cups flour
$1\frac{1}{2}$ cups flour	2 eggs	
	$\frac{3}{4}$ pound of chocolate chips	
Serves 10 people	Serves 60 people	Serves 20 people

Use your new recipe cards to find the total amount of each ingredient needed. Use the table below to help you.

Ingredient	Recipe 1 + 2 + 3 (Don't forget to find common denominators before adding.)	Total needed (Be sure to simplify any fractions.)
Flour		
Sugar		
Butter		
Vanilla		
Baking Soda		
Eggs		

When taking inventory in the pantry, you found that you already have some of the ingredients. Use the following table to organize your work. Don't forget common denominators.

Ingredient	Total needed from above	Already in Panty	Needs to be bought
Flour		$3\frac{1}{2}$ cups	
Sugar		2 cups	
Butter		$\frac{3}{4}$ cup	
Vanilla		2 teaspoons	
Baking Soda		$1\frac{1}{2}$ teaspoons	
Eggs		2	

3.1 Intro to Decimals

Place Value

The word form, decimal form, and fraction equivalent are shown here

One Hundred	Ten	One	One Tenth	One Hundredth	One Thousandth
100	10	1	0.1	0.01	0.001
$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Example: The number 132.524 would look like

Hundreds	Tens	Ones	Decimal Point	Tenths	Hundredths	Thousandths
1	3	2	.	4	2	4

This would be equivalent to the fraction $132\frac{524}{1000}$

We'd read this by reading the whole number, then the fraction equivalent

One hundred thirty two and four hundred twenty four thousandths.

Example: What is the place value of 4 in 65.413? The 4 is in the tenths place

Example: Write as a decimal: twenty three and forty six hundreds. 23.46

Converting a decimal to a fraction

To convert a decimal to a fraction, we write the decimal part as a fraction, then reduce if possible.

Example: Write 7.25 as a mixed number $7.25 = 7\frac{25}{100} = 7\frac{1}{4}$

Example: Write 5.4 as a mixed number $5.4 = 5\frac{4}{10} = 5\frac{2}{5}$

Rounding

When we round to a decimal place value, we look to the right of the desired place value to determine which way to round. Everything after the desired place value gets dropped.

Example: Round 173.264 to the nearest tenth

The 2 is in the tenths place. Looking to the right, the 6 tells us to round up, so we round to 173.3

Example: Round 173.264 to the nearest tenth

The 2 is in the tenths place. Looking to the right, the 6 tells us to round up, so we round to 173.3

Worksheet – 3.1 Intro to Decimals Name: ______________________________

1) Write out in words: 5.46

2) Write out in words: 7.912

3) Write the number: twenty three and five tenths

4) Write out the number: two thousand eleven and four hundred twenty six thousandths

5) What is the place value of 8 in 7.0812?

6) What is the place value of 2 in 7.0812?

8) Round 15.194 to the nearest tenth

9) Round 8.724 to the nearest whole number

10) Round 8.07 to the nearest tenth

11) Round 5.197 to the nearest hundredth

Determine which number is larger. Write < or > between the numbers to show this.

12) 4.512 4.508

13) 6.17 6.2

Convert to mixed numbers. Reduce to lowest terms.

14) 5.6

15) 7.12

16) 6.375

Measure the length of each bar in centimeters. Give your answer as a decimal.

17)

18)

19)

3.2 Adding and Subtracting Decimals

To add and subtract decimals, stack the numbers, aligning the place values and the decimal point. Add like place values, carrying as needed. The decimal point in the sum will be aligned with the decimal point in the numbers being added

Example: Add 3.15 and 5.38

$$\begin{array}{r} 3.15 \\ +\ 5.38 \\ \hline 8.53 \end{array}$$

If one decimal has more decimal places than another, you can optionally write additional zeros on the number with less decimal places, since that doesn't change the value of the number.

Example: Add 12.302 and 5.4

$$\begin{array}{r} 12.302 \\ +\ 5.4 \\ \hline \end{array} \qquad \text{Writing additional zeros:} \quad \begin{array}{r} 12.302 \\ +\ 5.400 \\ \hline 17.702 \end{array}$$

Subtraction works the same way, but it is more important here to write the additional zeros if the top number has less decimal places than the bottom number.

Example: Subtract 8.3 - 4.721

$$\begin{array}{r} 8.3 \\ -\ 4.721 \\ \hline \end{array} \qquad \begin{array}{r} 8.300 \\ -\ 4.721 \\ \hline \end{array} \qquad \begin{array}{r} 8.29{}^{1}0 \\ -\ 4.72\ 1 \\ \hline \end{array} \qquad \begin{array}{r} 7.{}^{1}29{}^{1}0 \\ -\ 4.\ 72\ 1 \\ \hline 3.\ 57\ 9 \end{array}$$

Example: Ariel has a balance of \$450.23 in her checking account. After paying an electric bill for \$57.50 and a cell phone bill for \$83.24, how much will she have left in her account?

We might start by adding the two bills:

$$\begin{array}{r} 57.50 \\ +83.24 \\ \hline 140.74 \end{array}$$

Now, subtracting this from \$450.23:

$$\begin{array}{r} 450.23 \\ -140.74 \\ \hline \end{array} \qquad \begin{array}{r} 450.1{}^{1}3 \\ -140.7\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 449.{}^{1}1{}^{1}3 \\ -140.\ 7\ 4 \\ \hline 309.\ 4\ 9 \end{array}$$

Ariel will have \$309.49 remaining in her account.

Worksheet – 3.2 Add / Subtract Decimals Name: ________________________________

Calculate.

1) 2.4 + 6.8

2) 3.05 + 1.4

3) 125.105 + 6.7

4) 9.8 – 4.2

5) 137.25 – 14.42

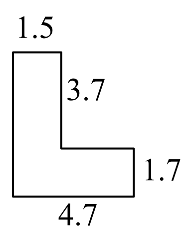
6) 8.1467 – 7.3

7) 10.3 – 12.135

8) 12.25 + 6.15 + 3.71

9) 10 – 7.27

11) Find the perimeter of the shape shown.



12) Estimate the value of the following sum by first rounding each value to the nearest hundredth:

12.916273 + 5.1 + 7.283461

3.3.1 Multiplying Decimals

Multiplying Decimals

To multiply decimals, line up the numbers on the right side. There is no need to add additional zeros if the decimals have different lengths. Multiply the two numbers, ignoring the decimal points. To place the decimal point in the answer, count up the number of decimal places in each number you're multiplying; the answer will have that many decimal places.

Example: Multiply 3.15 times 6.4

```
   3.15  <- Has 2 decimal places
  x 6.4  <- Has 1 decimal place
  1.260  <- Needs 3 decimal places
+18.900  <- Needs 3 decimal places
 20.160  <- Answer has 3 decimal places
```

Example: Multiply 12.5 times 0.013

```
   12.5  <- Has 1 decimal place
x 0.013  <- Has 3 decimal places
  .0375  <- Needs 4 decimal places
  .1250  <- Needs 4 decimal places
 0.1625  <- Answer has 4 decimal places
```

Multiplying by multiples of 10

Example: Multiply 5.9134 times 1000

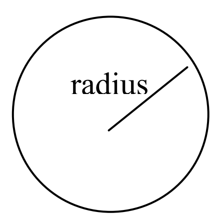
```
   5.9134  <- Has 4 decimal places
  x   1000  <- Has 0 decimal places
5913.4000  <- Answer has 4 decimal places
```

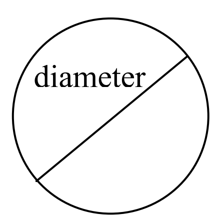
Notice that multiplying the decimal by 1000 had the effect of moving the decimal places 3 places to the right – the same as the number of zeros in 1000.

Area and Perimeter of Circles

The **area of a circle** can be found using the formula $A = \pi r^2$, where r is the radius of the circle, and π is a symbol representing a number. π is approximately 3.141592654, but for our purposes, we'll round to 3.14. Perimeter of a circle is also called **circumference**, and can be found using the formula $C = 2\pi r$.

Example: Find the area of a circle with radius 4 feet.





The area is $\pi \cdot 4^2$, which we'll approximate by calculating $3.14(4^2)$

$3.14(4)^2 = 3.14(16)$

```
   3.14  <- Has 2 decimal places
  x  16  <- Has 0 decimal places
  18.84  <- Needs 2 decimal places
  31.40  <- Needs 2 decimal places
  50.24
```

The area of the circle is approximately 50.24 square feet.

Worksheet – 3.3.1 Multiplying Decimals Name: ________________________________

Multiply

1) 7(4.6)

2) (8.3)(12.4)

3) (3.04)(0.02)

4) (2045)(0.04)

5) (100)(1.623)

6) (0.03)(0.14)

7) (1.004)(0.87)

8) (1.3)(2.5)(10)

9) (7.3)(0.021)

10) Find the area of a rectangle 10.2 meters wide and 8.7 meters tall

12) Find the area and circumference of a circle with radius 6 cm.

13) Find the area and circumference of a circle with diameter 10 inches.

3.3.2 Dividing Decimals

To divide a decimal by a whole number, divide as usual, putting the decimal place in the result directly above the decimal point in the number you're dividing.

Example: Divide $7.12 \div 4$

```
   1.         1.7         1.78
4)7.12     4)7.12      4)7.12
 -4         -4          -4
  3.1        3.1         3.1
            -2.8        -2.8
             0.32        0.32
                        -0.32
                            0
```

Sometimes it is necessary to add additional zeros to the end of the number you're dividing. You can continue adding zeros as needed.

Example: Divide $5 \div 8$

```
   0.         0.6         0.62          0.625
8)5.0      8)5.00      8)5.000      8)5.000
 -0         -0          -0           -0
  5.0        5.0         5.0          5.0
            -4.8        -4.8         -4.8
             0.20        0.20         0.20
                        -0.16        -0.16
                         0.040        0.040
                                     -0.040
                                      0.000
```

To divide two decimals, it is sometimes easier to multiply both decimals first by 10, 100, 1000, etc. so that the divisor is a whole number

Example: Divide $1.42 \div 0.3$

We can first multiply both numbers by 10 to create the equivalent problem $14.2 \div 3$. Notice this is equivalent to moving the decimal place in both numbers one place to the right.

```
    4          4.7          4.73          4.733
3)14.2     3)14.20     3)14.200      3)14.200
 -12        -12         -12           -12
   2.2        2.2         2.2           2.2
             -2.1        -2.1          -2.1
              0.10        0.10          0.10
                         -0.09         -0.09
                          0.010         0.010
                                       -0.009
                                        0.001
```

At this point, you might notice that the pattern continues, and the answer would be 4.73333333333333… continuing on forever. We shorthand this writing a bar above the 3 to indicate that it repeats: $4.7\overline{3}$

This process for dividing decimals is also how we can convert a fraction to a decimal.

Worksheet – 3.3.2 Dividing Decimals Name: ________________________________

Divide

1) $7\overline{)24.64}$

2) $3.2\overline{)26.08}$

3) $0.4\overline{)9}$

4) $6.84 \div 0.002$

5) $130 \div 2.5$

6) $\frac{9.03}{2.1}$

Convert each fraction into a decimal by dividing

7) $\frac{2}{5}$

8) $\frac{3}{8}$

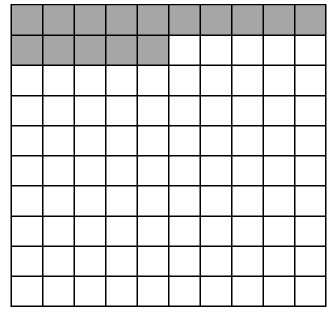
9) $\frac{5}{6}$

10) Four roommates need to split a \$52.68 electric bill evenly. How much will each need to pay?

3.4 Intro to Percents

What is a percent

Percent means “per hundred” or “out of 100.” The symbol % is used after a number to indicate a percent.



Example: 15% means 15 out of 100, or $\frac{15}{100}$ as fraction. Visually, the box to the right has 15% of the squares shaded: 15 out of the 100.

Writing a percent as a decimal or fraction

To write a percent as a fraction, write the percent as a fraction of 100.

Example: Write 23% as a fraction. $23\% = \frac{23}{100}$

Example: Write 50% as a fraction. $50\% = \frac{50}{100} = \frac{1}{2}$

To write a decimal as a percent, look to see how many hundredths you have

Example: Write 0.4 as a percent. $0.4 = 0.40 = \frac{40}{100} = 40\%$

You may notice this is the same as moving the decimal place to the right two places
Example: Write 0.057 as a percent. Moving the decimal to the right two places: 5.7%

To convert a percent to a decimal, write the decimal out of 100 and divide. Notice this is the same as moving the decimal place to the left two places.
Example: Write 12.5% as a decimal and as a fraction.
As a decimal, $12.5\% = \frac{12.5}{100} = 0.125$

To write as a fraction, we could start with the decimal: $0.125 = \frac{125}{1000} = \frac{1}{8}$

To write a fraction as a percent, first divide to find a decimal, then write it as a percent.

Example: Write $\frac{2}{5}$ as a percent. $2 \div 5 = 0.4 = 0.40 = 40\%$

To find the percent of a whole

Often we want to find the percent of a whole. To do this we multiply, first writing the percent as a decimal:
whole · percent = part

Example: Find 20% of 80. 80 is the whole. To find 20% of it, we convert 20% = 0.20 and multiply:
$80(0.20) = 16$. 20% of 80 is 16.

Example: If sales tax is 9.6%, find the sales tax on a \$200 purchase.
We would multiply: $200(0.096) = 19.2$. Tax would be \$19.20.

Worksheet – 3.4 Intro Percents Name: ________________________________

Rewrite as a decimal

1) 20% 2) 46% 3) 7.4% 4) 0.3% 5) 127%

Rewrite as a percent

6) 0.74 7) 0.9 8) 0.0254 9) 1.35 10) 0.05

Rewrite as a percent. Round to the nearest tenth of a percent if needed.

11) $\frac{3}{4}$ 12) $\frac{1}{2}$ 13) $\frac{1}{6}$ 14) $\frac{7}{10}$ 15) $\frac{47}{50}$

Rewrite as a reduced fraction

16) 0.25 17) 0.3 18) 0.05 19) 0.025

Find the desired percent

20) Find 15% of 60 21) Find 40% of 32 22) Find 5% of 56

23) If you want to leave a 20% tip on a $27.56, how much should you leave?

3.5 Solving Percent Problems

Pieces of a percent problem

Percent problems involve three quantities: the **base** amount (the whole), the **percent**, and the **amount** (a part of the whole). The amount is a percent of the base.

Example: 50% of 20 is 10. 20 is the base (the whole). 50% is the percent, and 10 is the amount (part of the whole)

In percent problems, one of these quantities will be unknown. Here are the three cases:

Example: What is 25% of 80? 80 is the base (the whole) we are finding a percent of. The percent is 25%. The amount is unknown.

Example: 60 is 40% of what number? The percent is 40%. The unknown is the base that we are finding a percent of. The amount (part of the whole) is 60.

Example: What percent of 320 is 80? The base we are finding a percent of is 320. The percent is unknown. The amount is 80.

Solving percent problems

To solve percent problems we use this relationship:

$$\boxed{\text{base} \cdot \text{percent} = \text{amount}}$$

Example: What is 25% of 80? The base is 80 and the percent is 25%, so amount = 80(0.25) = 20

Example: 60 is 40% of what number? The percent is 40%., the amount is 60, and the base is unknown. Using this, we can say

$base \cdot 0.4 = 60$ To solve for the base, we divide both sides by 0.4

$base = \dfrac{60}{0.4} = 150$ 60 is 40% of 150.

Example: What percent of 320 is 80? The base is 320, the amount is 80, and the percent is unknown. Using this, we can say

$320 \cdot percent = 80$ To solve for the percent, we divide both sides by 320

$percent = \dfrac{80}{320} = 0.25$ 25% of 320 is 80.

Example: An article says that 15% of a non-profit's donations, about $30,000 a year, comes from individual donors. What is the total amount of donations the non-profit receives?

The percent is 15%. $30,000 is the amount – a part of the whole. We are looking for the base.

$base \cdot 0.15 = 30{,}000$ To solve for the base, we divide both sides by 0.15

$base = \dfrac{30{,}000}{0.15} = \$200{,}000$ The non-profit receives $200,000 a year in donations

Worksheet – 3.5 Solving Percent Problems

Name: ________________________________

1) 30% of what number is 54

2) What number is 40% of 8?

3) What percent of 200 is 40?

4) 10 is 5% of what number?

5) What is 120% of 30?

6) $30 is what percent of $80?

7) Out of 300 diners, 60 ordered salads. What percent of diners ordered a salad?

8) The population of the US is around 300,000,000. How many people make up 1% of the population?

9) Bob bought a $800 TV on sale for $650. What percent savings is that? (be careful!)

10) Out of 200 people, 40 own dogs. What percent is that? Out of 550 people, how many would you expect to own dogs?

3.6 Percent Applications

Common uses of percents

Percents are commonly used in calculating taxes, discounts, markups, and commissions

Example: Marcus sold \$1200 worth of electronics last week, and earns a 5% commission. How much did he make last week?
We need to find 5% of 1200: 1200(0.05) = \$60

Often times these problems will require two steps
Example: A shirt was originally \$35 and is discounted 30% off. How much will it cost now?
First we find the discount. 30% of \$35 is \$35(0.30) = \$10.50. This is how much we save.
So the discounted price would be \$35 - \$10.50 = \$24.50

Example: A retailer adds a 40% markup on books. If they get the book wholesale for \$15, how much will it retail for after the markup is added?
First we find the markup. 40% of \$15 is \$15(0.40) = \$6. This is how much they add to the price.
So the retail price would be \$15 + \$6 = \$11

Example: On your paystub, you notice they withhold \$157.50 of your \$900 check for taxes. What percent are they withholding?
The base is \$900 and the amount is \$157.50. To find the percent we divide:
157.50 ÷ 900 = 0.175 = 17.5% is withheld

Example: The price of a movie ticket increased from \$8.50 to \$9.75 at a theater. What percent increase is that?
First we need to find the amount of increase: The price increased \$9.75 - \$8.50 = \$1.25.
Now we ask what percent of \$8.50 is \$1.25, since \$8.50 was the original price. We divide:
1.25 ÷ 8.5 is approximately 0.147 = 14.7%. The price increased about 15%.

Simple Interest

Interest is a fee for borrowing or lending money. Simple interest is calculated as a percent of the original amount borrowed (the **principal)**. The **interest rate** is usually per year, so **time** is also part of calculating interest.

$I = p \cdot r \cdot t$
p: the principal
I: interest paid
r: interest rate
t: time

Example: You borrow \$1,000 at 4% interest for 3 years. How much interest will you pay?
The principal is \$1,000, the interest rate is 4%, and the time is 3 years. Using the formula:
$I = p \cdot r \cdot t = (1000)(0.04)(3) = \$120$ in interest

Example: A loan of \$12,000 was made for 3 months charging 20% interest. Find the interest charge.

The principal is \$12,000, the interest rate is 20%, and the time is 3 months, which is $\frac{3}{12}$ years.

$I = p \cdot r \cdot t = (12000)(0.2)\left(\frac{3}{12}\right) = \$600$ in interest.

Worksheet – 3.6 Percent Applications Name: ________________________________

1) Joelle earned \$120 commission on \$2000 of sales. What percent commission does she earn?

2) A TV originally costing \$900 is on sale for 20% off. What is the sale price?

3) The unemployment rate rose from 7.7% in 2009 to 10% in 2010. What percent increase is that?

4) A used car costs \$3,500. How much will you have to pay after adding 9.5% sales tax?

5) A store marks up their electronics by 10%. If they buy a smartphone wholesale for \$260, how much will they sell it for?

6) Polly borrows \$300 at 5% interest for 2 years. How much interest will she pay?

7) Jordan buys a $16,000 car at 6% interest. How much interest will he pay in the first month?

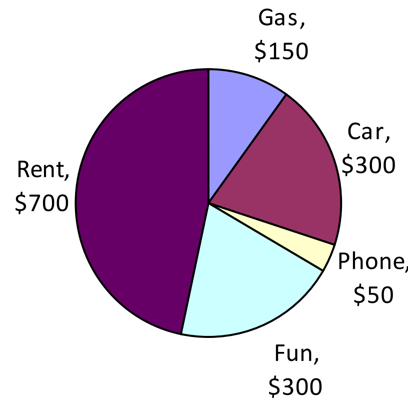
8) A payday loan charges $30 interest for a 1 month loan of $200. What is the interest rate?

9) A dress was originally $100, but is on sale for 10% off. How much will it be after adding 10% sales tax? (it's not $100)

10) After having 15% of his paycheck withheld for deductions, Marcus made $510. How much was his paycheck before deductions? (it's not $586.50)

3.7 Pie Charts

A **pie chart**, or **circle graph**, is commonly used to show how a large whole is broken up into smaller pieces. The whole is represented by a circle, with slices of the circle representing the pieces – the size of the slice corresponds with the size of the piece.

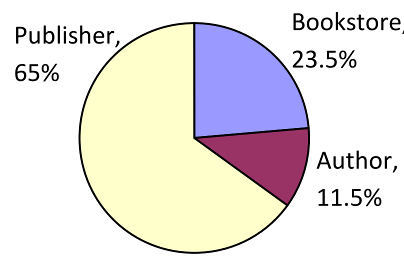


Reading pie charts

Example: The chart to the right shows James's budget for the month. What is his total budget for the month? What percent of his budget is he spending on his car?

Often pie charts show percentages rather than values

Example: The chart to the right shows where textbook dollars go. If you buy a $150 textbook, how much does the author get?



Creating pie charts

To create a pie chart, we first find the percentage of a whole each piece is, then draw slices that are approximately that percentage of the whole circle.

Example: A survey of 600 people, 300 said they were satisfied with their employment, 240 said they were looking for a better job, and 60 said they were currently unemployed.

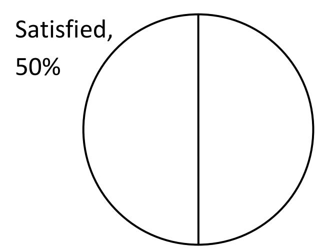
First we convert these results into percents:

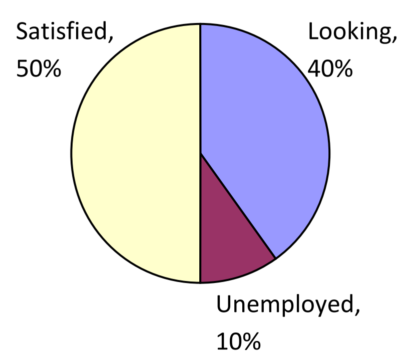
Satisfied: $\frac{300}{600} = 0.50 = 50\%$

Looking: $\frac{240}{600} = 0.40 = 40\%$

Unemployed: $\frac{60}{600} = 0.10 = 10\%$

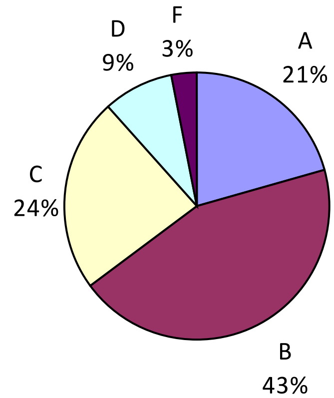
We now divide the circle up. The "Satisfied" slice should be 50%, which is half of the circle. The "Unemployed" slice should be 10%, or $\frac{1}{5}$ of the remaining half. We draw the slices, then label them both with what they represent, as well as the percent.

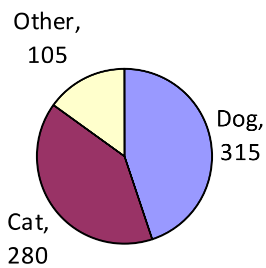




Worksheet – 3.7 Pie Charts Name: ________________________________

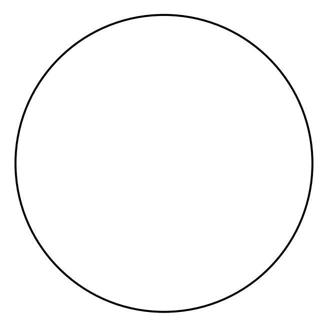
1) The chart to the right shows the grade distribution on a quiz in one of my classes. If there are 34 students in the class, how many scored an "A"?





2) The chart to the left shows the number of animals served at a vet clinic. What percentage of the animals served were dogs?

3) In a September Gallup poll, the people survey were asked if they felt Obama's jobs bill would help in creating new jobs. 27% said they felt it would help a lot; 38% said they felt it would help a little; 30% said they felt it would not help; 5% had no opinion. Create a pie chart to show this result.



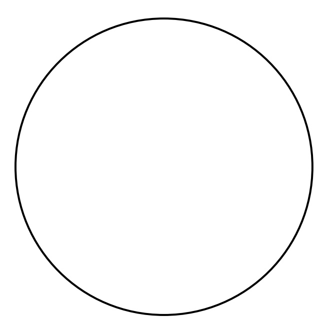
4) A group of people were asked their blood types. Create a pie chart to show this data.

O: 132

A: 126

B: 30

AB: 12



Percents Activity Names: ________________________________

Imagine the country is made up of 100 households. The federal government needs to collect $800,000 in income taxes to be able to function. The population consists of 5 groups:

Group A: 20 households that earn $12,000 each
Group B: 20 households that earn $29,000 each
Group C: 20 households that earn $50,000 each
Group D: 20 households that earn $79,000 each
Group E: 15 households that earn $129,000 each
Group F: 5 households that earn $295,000 each

We have been tasked to determine new income tax rates.

The first proposal we'll consider is a flat tax – one where every income group is taxed at the same percentage tax rate.

1) Determine the total income for the population (all 100 people together)

2) Determine what flat tax rate would be necessary to collect enough money.

The second proposal we'll consider is a modified flat-tax plan, where everyone only pays taxes on any income over $20,000. So, everyone in group A will pay no taxes. Everyone in group B will pay taxes only on $9,000.

3) Determine the total *taxable* income for the whole population

4) Determine what flat tax rate would be necessary to collect enough money in this modified system

5) Complete this table for both the plans

		Flat Tax Plan		Modified Flat Tax Plan	
Group	**Income per household**	**Income tax per household**	**Income after taxes**	**Income tax per household**	**Income after taxes**
A	$12,000				
B	$29,000				
C	$50,000				
D	$79,000				
E	$129,000				
F	$295,000				

The third proposal we'll consider is a progressive tax, where lower income groups are taxed at a lower percent rate, and higher income groups are taxed at a higher percent rate. For simplicity, we're going to assume that a household is taxed at the same rate on all their income.

6) Set progressive tax rates for each income group to bring in enough money. There is no one right answer here – just make sure you bring in enough money!

Group	Income per household	Tax rate (%)	Income tax per household	Total tax collected for all households	Income after taxes per household
A	$12,000				
B	$29,000				
C	$50,000				
D	$79,000				
E	$129,000				
F	$295,000				
				This better total to $800,000	

7) Which plan seems the most fair to you? Which plan seems the least fair to you? Why?

Quintile incomes rounded from http://www.taxpolicycenter.org/taxfacts/displayafact.cfm?Docid=330
$$ needed from http://en.wikipedia.org/wiki/File:U.S._Federal_Receipts_-_FY_2007.png, divided by number of households from http://www.census.gov/population/projections/nation/hh-fam/table1n.txt

4.1 Rates and Ratios

Ratios are used to compare amounts or describe a relationship between two quantities, usually of the same type.

Example: Out of 300 students in the class, 175 are female. Express this as a ratio.

The ratio of female students to total students is $\frac{175}{300}$. This is often read "175 out of 300". In this case, this ratio could be expressed in lower terms as $\frac{7}{12}$.

Sometimes you'll see this written as "7 to 12" or "7:12"

Example: A rectangle is 14 inches long and 8 inches wide. What is the ratio of length to width?

The ratio is $\frac{14}{8} = \frac{7}{4}$

A **rate** is a ratio that compares quantities with different units. Examples include miles per hour, miles per gallon, dollars per hour, cost per ounce, etc. As with ratios, we usually reduce rates to lowest terms.

Example: Write the rate as a simplified fraction: 6 teachers for 63 students.

$$\frac{6\text{ teachers}}{63\text{ students}} = \frac{2\text{ teachers}}{31\text{ students}}$$

Example: Write the rate as a simplified fraction: With 5 gallons of gas you drive 150 miles

$$\frac{5\text{ gallons}}{150\text{ miles}} = \frac{1\text{ gallon}}{30\text{ miles}}$$

A **unit rate** is a rate where the denominator is 1. To accomplish this, we may end up with a fraction or decimal in the numerator.

Example: A 24 ounce bottle of shampoo sells for \$3.60. Find the unit rate (unit cost: cost per ounce)

$$\frac{3.60\text{ dollars}}{24\text{ ounces}} = \frac{3.60\text{ dollars} \div 24}{24\text{ ounces} \div 24} = \frac{0.15\text{ dollars}}{1\text{ ounce}}$$

We would typically read this unit rate as "15 cents <u>per ounce</u>"

Example: Kate worked 32 hours last week and earned \$304. Find her unit pay rate (dollars per hour).

We are looking for dollars per hour, so dollars must go in the numerator, and hours in the denominator.

$$\frac{304\text{ dollars}}{32\text{ hours}} = \frac{304\text{ dollars} \div 32}{32\text{ hours} \div 32} = \frac{9.5\text{ dollars}}{1\text{ hour}}$$

Kate earns \$9.50 per hour.

Worksheet – 4.1 Intro to Ratios and Rates Name: _______________________________

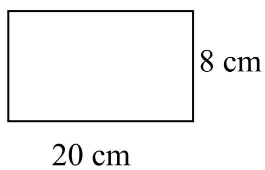
Write each ratio as a fraction, simplifying if possible.

1) 6 pounds to 15 pounds

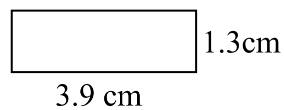
2) Jake make 24 free throws out of 32 attempts

For each rectangle shown, write the ratio of the longer side to the shorter side. Simplify.

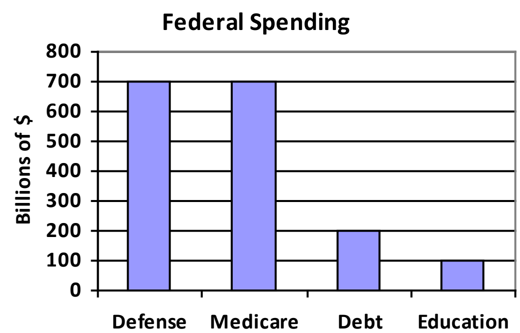
3)



4)



5) From the chart shown, find the ratio of spending on Medicare to spending on debt interest.



Write each rate in lowest terms

6) 30 cookies for 8 people

7) 260 miles in 4 hours

Find each unit rate

8) $90 earned in 8 hours

9) 32 pounds of zucchini from 5 plants

10) $340 for 0.2 ounces of gold

11) $9.75 for 12 cans

12) Which is a better deal: 24 oz box of cereal for $3.79 or a 32 oz box for $4.89?

4.2 Proportions

A **proportion** is an equation showing the equivalence of two ratios or rates.

Example: Is this proportion true? $\frac{\text{2 cups flour}}{\text{30 cookies}} = \frac{\text{4 cups flour}}{\text{60 cookies}}$

Yes, this proportion is true since the two fractions are equivalent since $\frac{2}{30} = \frac{1}{15} = \frac{4}{60}$, and both rates have the same units of "cups flour per cookies."

Example: Is this proportion true? $\frac{\text{2 gallons}}{\text{50 miles}} = \frac{\text{100 miles}}{\text{4 gallons}}$

This proportion cannot be true since the rate on the left has units "gallons per mile" and the rate on the right has units "miles per gallon". Since the units are different, these are not comparable.

Solving proportions

If a proportion is true, then if we **cross-multiply**, both sides will be equal.

Example: $\frac{4}{5} = \frac{8}{10}$ is a true proportion since $4 \cdot 10 = 8 \cdot 5$

$$\frac{4}{5} = \frac{8}{10}$$

To solve for an unknown in a proportion, we can cross-multiply, then divide.

Example: Solve for the unknown *n*. $\frac{n}{20} = \frac{12}{80}$

Cross multiplying, $n \cdot 80 = 20 \cdot 12$, or $80n = 240$. Dividing, $n = \frac{240}{80} = 3$.

Example: A picture is taken that is 4 inches tall and 6 inches width. If you want to enlarge the photo to be 10 inches tall, how wide will it be?

We can set up a proportion, where both ratios have the same units: $\frac{\text{4 inches tall}}{\text{6 inches wide}} = \frac{\text{10 inches tall}}{x \text{ inches wide}}$.

Solving for the unknown *x*, we cross multiply:

$4 \cdot x = 6 \cdot 10$, or $4x = 60$. Dividing, $x = \frac{60}{4} = 15$ inches wide.

Example: A report shows 2 out of 3 students receive some financial aid. Out of 1200 students, how many would you expect to be receiving financial aid?

We can set up a proportion: $\frac{\text{2 students with financial aid}}{\text{3 students total}} = \frac{n \text{ students with financial aid}}{\text{1200 students total}}$.

$2 \cdot 1200 = 3 \cdot n$, or $2400 = 3n$. Dividing, $n = \frac{2400}{3} = 800$ students receiving financial aid.

Notice that this is exactly the same as finding 2/3 of 1200, or 66.67% of 1200.

Worksheet – 4.2 Proportions Name: ________________________________

Is each proportion true?

1) $\dfrac{\$3}{12\text{ eggs}} = \dfrac{\$4.50}{18\text{ eggs}}$ 2) $\dfrac{24\text{ trees}}{2\text{ acres}} = \dfrac{48\text{ acres}}{4\text{ trees}}$ 3) $\dfrac{18}{6} = \dfrac{45}{15}$ 4) $\dfrac{5.4}{8} = \dfrac{6.8}{10}$

Solve the proportion for the unknown

5) $\dfrac{n}{6} = \dfrac{10}{15}$ 6) $\dfrac{32}{4} = \dfrac{r}{11}$ 7) $\dfrac{49}{q} = \dfrac{7}{9}$ 8) $\dfrac{20}{8} = \dfrac{5}{x}$

9) $\dfrac{n}{5} = \dfrac{10}{8}$ 10) $\dfrac{8}{2/3} = \dfrac{9}{x}$ 11) $\dfrac{3\frac{1}{2}}{7} = \dfrac{n}{2\frac{1}{3}}$ 12) $\dfrac{8.7}{n} = \dfrac{0.4}{0.2}$

13) If 6 ounces noodles makes 3 servings, how many ounces of noodles do you need for 8 servings?

14) If you are supposed to mix 3 ounces of floor cleaner to every 2 cups of water, how much water should you mix with 8 ounces of cleaner?

15) At 3pm, Mikayla’s shadow was 1.2 meters long. Mikayla is 1.7 meters tall. She measures a tree’s shadow to be 7.5 meters. How tall is the tree?

Proportional Geometry

Two shapes are **similar** or **proportional** when the ratio of their sides is proportional.

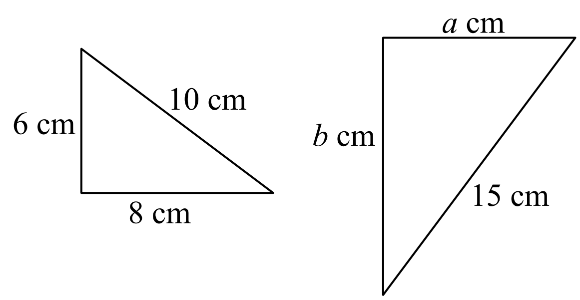
Example: A photo is 4 inches tall and 6 inches wide. To be proportional, how tall would the photo need to be if it were 10 inches wide?

We can set up a proportion of $\frac{\text{tall}}{\text{wide}}$: $\frac{4 \text{ inches}}{6 \text{ inches}} = \frac{x \text{ inches}}{10 \text{ inches}}$

Cross multiplying, $4 \cdot 10 = 6 \cdot x$, or $40 = 6x$. Dividing, $x = \frac{40}{6} = 6\frac{2}{3}$ inches.

Example: Given the triangle shown, find the unknown sides of the similar triangle

Notice the triangle has been rotated. The slanted side of length 10 in the original triangle corresponds with the slanted side of length 15 on the larger triangle. Likewise, the shortest side 6 on the original will correspond with the shortest side *a* on the larger triangle. We can set up two proportions:



$\frac{6 \text{ cm}}{10 \text{ cm}} = \frac{a \text{ cm}}{15 \text{ cm}}$, so $10a = 90$. $a = 9$

$\frac{8 \text{ cm}}{10 \text{ cm}} = \frac{b \text{ cm}}{15 \text{ cm}}$, so $10b = 120$. $b = 12$

Note: It would have been equally correct to have set up the first ratio as $\frac{6 \text{ cm}}{a \text{ cm}} = \frac{10 \text{ cm}}{15 \text{ cm}}$

Example: In a map, a scale is often given. For example, 0.5 inch on the map might correspond to 5 miles in real life. If two objects are 2.3 inches apart on the map, how far apart are they in real life?

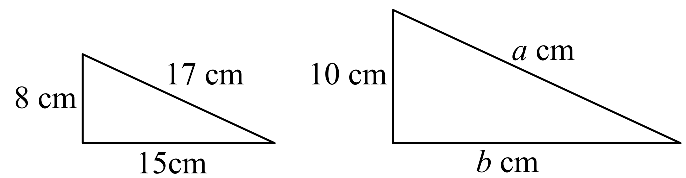
We can set up a proportion: $\frac{0.5 \text{ inches}}{5 \text{ miles}} = \frac{2.3 \text{ inches}}{x \text{ miles}}$

Cross multiplying, $0.5 \cdot x = 2.3 \cdot 5$, or $0.5x = 11.5$. Dividing, $x = \frac{11.5}{0.5} = 23$ miles

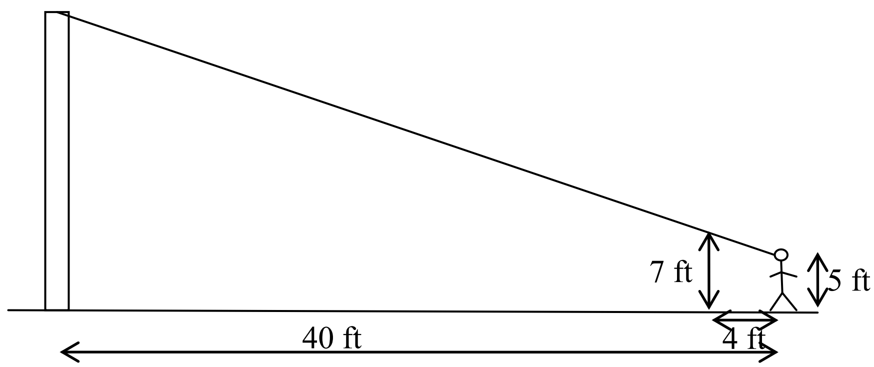
Worksheet – Proportional Geometry Name: ___________________________________

1) A standard wheelchair ramp should have a grade no steeper than 1:12. This means that for every inch of rise (change in height), there should be 12 inches of run (change in length). If a door is $2\frac{1}{2}$ feet above the ground, how long would the wheelchair ramp need to be?

2) Find the unknown sides



3) To find the height of a tower 40 feet away, you hang a tape measure 4 feet from you and look up to the top of the tower. Your eyes are 5 feet above the ground. The top of the tower lines up with a point on the tape measure 7 feet off the ground. How tall is the tower?



4) A model car is a 1:18 scale, meaning that 1 inch on the model is 18 inches in the real car. If the model is 11 inches long, how long is the actual car?

5) Use the scale in the map shown to find the distance between Austin and San Antonio.

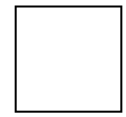
4.3 Volumes

Length measures distance in a line: The distance between two places, the length of your , etc. It is measured in units like inches, centimeters, feet, etc.

1 cm

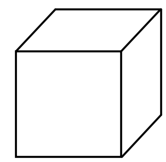
Area measures the amount of flat space a shape covers: The area of a floor, the size of a city, etc. It is measured in square units, like square inches, square centimeters, square feet, etc.

1 cm^2



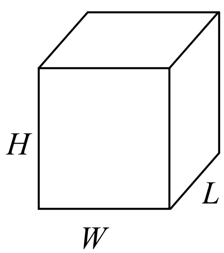
Volume measures the amount of three-dimensional space a shape fills: The amount of water in the bathtub, the number of gallons of gas in your gas tank, the size of a brick, etc. It is measured in cubic units, like cubic inches, cubic centimeters, cubic feet, etc.

1 cm^3

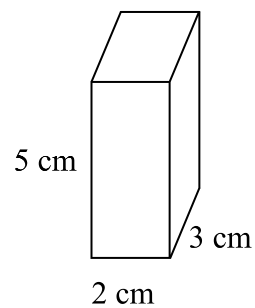


Volumes of basic shapes

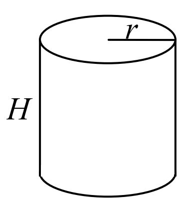
Rectangular boxes
L = length, W = width, H = height
Volume $= L \cdot W \cdot H$



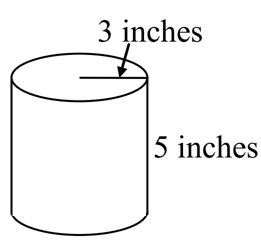
Example:
Volume = (5 cm)(2 cm)(3 cm) = 30 cm^3



Cylinders
r = radius, H = height
Volume $= \pi r^2 H$



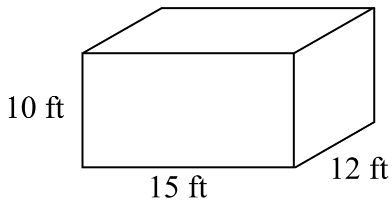
Example:
Volume = $\pi(3)^2 5 = \pi \cdot 45 = 141.3$ inches^3



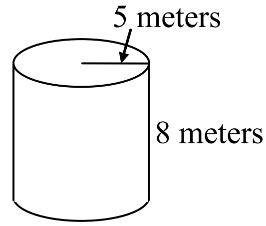
Worksheet – 4.3 Volumes   Name: ________________________________

Find the volume of each shape

1)



2)



3) How much water does a swimming pool 20 meters long, 5 meters wide, and 3 meters deep hold?

4) A grain silo is shaped like a cylinder. It has a *diameter* of 20 feet and is 18 feet tall. Find the volume.

5) A cereal box previously originally was 12 inches by 8 inches by 3 inches. To save money, they reduced each dimension by 0.5 inches. How much did the volume change?

6) Sami needs to order concrete for a driveway. The driveway will be 12 feet wide, 20 feet long, and ¼ foot thick. How many cubic feet of concrete will Sami need to order?

4.4 Converting Units – U.S. Units

Converting between units, it can be tricky to determine if you need to multiply or divide. To make this easier, we're going to use **dimensional analysis**, which is just a fancy way of keeping track of units. To convert units, we multiply by a **conversion factor** – a rate that where the numerator is equal to the denominator.

To convert from feet to inches we can use the conversion factor $\frac{12\text{ inches}}{1\text{ foot}}$ or $\frac{1\text{ foot}}{12\text{ inches}}$, since 1 ft = 12 inches. When we multiply by a conversion factor, the units cancel like numbers, so we multiply by the conversion factor that will cancel the way we want.

Example: Convert 30 inches to feet.

To make this conversion, we notice we have 30 inches, or $\frac{30\text{ inches}}{1}$ as a fraction. To cancel the inches, we would need to multiply by $\frac{1\text{ foot}}{12\text{ inches}}$: $\frac{30\text{ inches}}{1}\cdot\frac{1\text{ foot}}{12\text{ inches}}=\frac{30\text{ feet}}{12}=2\frac{1}{2}$ feet.
Notice that the inches canceled, leaving feet.

Example: Convert $4\frac{1}{3}$ yards to feet.

3 feet = 1 yard. Since we want to eliminate yards, we'll put that in the denominator: $\frac{3\text{ feet}}{1\text{ yard}}$

$$4\frac{1}{3}\text{ yards}\cdot\frac{3\text{ feet}}{1\text{ yard}}=\frac{13\text{ yards}}{3}\cdot\frac{3\text{ feet}}{1\text{ yard}}=\frac{13\text{ feet}}{1}=13\text{ feet}$$

Example: Convert 4.5 pounds to ounces

1 pound = 16 ounces. To cancel the pounds, we'll use $\frac{16\text{ ounces}}{1\text{ pound}}$

$$\frac{4.5\text{ pounds}}{1}\cdot\frac{16\text{ ounces}}{1\text{ pound}}=\frac{72\text{ ounces}}{1}=72\text{ ounces}$$

Example: How many quarts is 20 cups?

1 quart = 4 cups. Since we want to eliminate cups, we'll use $\frac{1\text{ quart}}{4\text{ cups}}$

$$\frac{20\text{ cups}}{1}\cdot\frac{1\text{ quart}}{4\text{ cups}}=5\text{ quarts}$$

Example: How many fluid ounces is 1 gallon?
We don't have a conversion between these, but we can convert from gallons to cups, then cups to fluid ounces.

$$\frac{1\text{ gallon}}{1}\cdot\frac{16\text{ cups}}{1\text{ gallon}}\cdot\frac{8\text{ fl oz}}{1\text{ cup}}=128\text{ fluid ounces}$$

Worksheet – 4.4 Converting units U.S. Name: ______________________________

Length
1 foot = 12 inches
1 yard = 3 feet
1 mile = 5,280 feet

Weight
1 pound = 16 ounces
1 ton = 2000 pounds

Capacity
1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints = 4 cups
1 gallon = 4 quarts = 16 cups

3 teaspoon = 1 tablespoon
4 tablespoons = ¼ cup

Time
1 week = 7 days
60 minutes = 1 hour
60 seconds = 1 minute
24 hours = 1 day

Convert:

1) An elephant weighting 4.5 tons, to pounds

2) 43296 feet to miles

3) 4 fl oz to cups

4) 2 cups to tablespoons

5) 3 weeks to days

6) 3 weeks to hours

7) ¼ mile to yards

8) If your car emits 30 pounds of carbon dioxide every day, how many tons does it emit in a year?

9) 1 cubic foot is approximately 7.5 gallons. How many gallons would be needed to fill a pool 10 feet long, 8 feet wide, and 6 feet deep?

10) A bottle of cough syrup holds 12 fluid ounces. One dose is 1 tablespoon. How many doses are in the bottle?

Worksheet – 4.5 Converting units metric Name: ________________________________

Converting units in metric has the advantage that everything is multiplying or dividing by powers of 10, we can just move the decimal place.

Example: Convert 24 centimeters to meters

$$\frac{24\,\text{cm}}{1}\cdot\frac{1\,\text{meter}}{100\,\text{cm}}=\frac{24\,\text{meters}}{100}=0.24\text{ meters}$$

Length
1000 millimeters (mm) = 1 meter
100 centimeters (cm) = 1 meter
1000 meters (m) = 1 kilometer (km)

Mass
1000 milligrams (mg) = 1 gram (g)
1000 grams = 1kilogram (kg)

Capacity
1000 milliliters (ml) = 1 liter (L)

Less used:
100 centileters = 1 liter
10 deciliters = 1 liter
10 liters = 1 dekaliter
100 liters = 1 hectoliter

Convert:

1) 32 meters to kilometers

2) 0.3 grams to milligrams

3) 2.41 km to meters

4) 1.4 cm to millimeters

5) 120 ml to liters

6) 0.04kg to milligrams

7) If you take 4 pills, each containing 350 mg, how many grams of medicine total have you taken?

8) You have a rope 4 meters long, and need to cut pieces 30 cm long for an art project. How many pieces can you cut out of the rope?

Converting Rates

In more complicated conversions, we may need to do multiple conversions.   To convert, we multiply by a **conversion factors** –rates where the numerator is equal to the denominator.

Example: Convert 10 miles per hour to feet per minute.

To make this conversion, we can write 10 miles per hour as a fraction: $\frac{30 \text{ miles}}{1 \text{ hour}}$ as a fraction.

To convert the miles to feet, we would need to cancel feet and get miles. Since the original has miles in the numerator, our conversion factor needs miles in the denominator: $\frac{5280 \text{ feet}}{1 \text{ mile}}$

To convert the hours to minutes, we would need to cancel hours and get minutes. Since the original has hours in the denominator, our conversion factor needs hours in the numerator: $\frac{1 \text{ hour}}{60 \text{ minutes}}$

Multiplying, notice all the units cancel except feet on the top, and minutes on the bottom:

$$\frac{30 \cancel{\text{ miles}}}{1 \cancel{\text{ hour}}} \cdot \frac{5280 \text{ feet}}{1 \cancel{\text{ mile}}} \cdot \frac{1 \cancel{\text{ hour}}}{60 \text{ minutes}} = \frac{158400 \text{ feet}}{60 \text{ minutes}} = \frac{2640 \text{ feet}}{1 \text{ minute}} = 2640 \text{ feet per minute}$$

We can also user this process to make conversions when we're given additional information.

Example: An acre of land can produce 6000 kilocalories per day. If 1 person requires 2000 kilocalories per day, how many people per day can be fed on 10 acres of land?

While this problem could be solved in other ways, we're going to use unit conversions.

We start with 10 acres of land: $\frac{10 \text{ acres}}{1}$.

To convert this to kilocalories, we use 1 acre = 6000 kilocalories. Since the original has acres in the numerator , to cancel it, we'll write this conversion with acres in the denominator: $\frac{6000 \text{ kilocalories}}{1 \text{ acres}}$

To convert to people, we use 1 person = 2000 kilocalories. To cancel kilocalories, this conversion will need kilocalories in the denominator: $\frac{1 \text{ person}}{2000 \text{ kilocalories}}$

Multiplying: $\frac{10 \cancel{\text{ acres}}}{1} \cdot \frac{6000 \cancel{\text{ kilocalories}}}{1 \cancel{\text{ acres}}} \cdot \frac{1 \text{ person}}{2000 \cancel{\text{ kilocalories}}} = \frac{6000 \text{ people}}{2000} = 3 \text{ people}$

Notice that we set things up so all the units would cancel, except for people

Worksheet – Converting Rates Name: ________________________________

1) Convert 25 feet per second to miles per hour

2) Convert 1/4 cup of soap per gallon water to fluid ounces of soap per quart of water

3) Convert 20 grams per centimeter to kilograms per meter

4) 1 inch = 2.54 centimeters. Use this to convert 5 feet to meters.

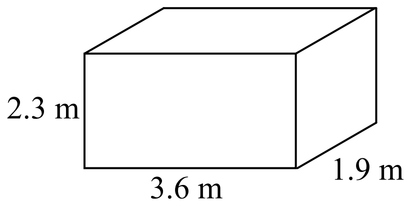
5) 1 penny has mass 4 grams, and is 1.55 millimeters thick. How much mass will a stack of pennies 20 centimeters tall have?

6) Electricity costs 6 cents per kilowatt-hour (that's 4 cents for 1 kilowatt used for 1hour). A Chevy Volt takes 10 hours to charge on a standard outlet, and draws about 2 kilowatts per hour. Find the cost to charge the car.

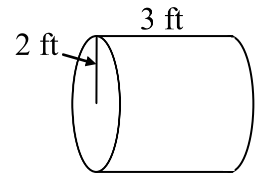
Worksheet – Chp 4 Review   Name: ________________________________

Find the volume of each shape

1)



2)



Is the proportion true?

3) $\frac{5.4}{9} = \frac{3}{5}$

Solve for the unknown

4) $\frac{6}{x} = \frac{4}{9}$

5) $\frac{n}{8} = \frac{2\frac{1}{4}}{10}$

6) $\frac{10}{4.3} = \frac{7}{a}$

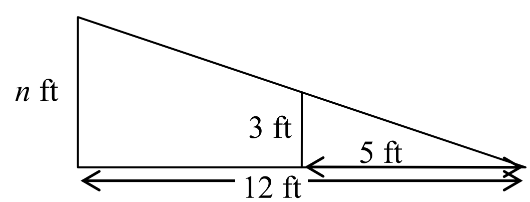
7) You can buy 4 pounds of bulk candy for \$3.40

a) Find the unit rate

b) How many pounds could \$6 buy you?

8) If a blood test is incorrect 1 out of 50 times, how many incorrect results would you expect if you test 3000 people?

9) Find the unknown height



Convert

10) 1.5 pounds to ounces

11) 1320 feet to miles

12) 0.7 km to meters

13) 14500 ml to liters

14) 80 ounces per cup to pounds per quart

15) US regulations recommend stairs should be 11 inches long and at most 7 inches tall. If a flight of stairs needs to go up 8 feet, how long will it need to be?

16) A recipe calls for 2 tablespoons of sugar for a batch that makes 30 pancakes. If you want to make a batch of 20 pancakes, how much sugar should you use? Convert to teaspoons.

17) The current exchange rate is 0.74 Euro (€) = 1 U.S. dollar. If something costs 40€, what is the price in US dollars?

18) While on vacation, you see this sign, showing gas for 1.276 Euro per liter. Knowing the exchange rate above, and that 1 gallon ≈ 3.8 L, find the price in dollars per gallon.

Hot Coffee Activity Name: ________________________

To accompany the materials at http://threeacts.mrmeyer.com/hotcoffee/

1) How big is the coffee cup?

2) How many gallons will it hold?

3) How many cups of coffee is that?

4) Will that break the record? By how much?

5) How long will it take to fill?

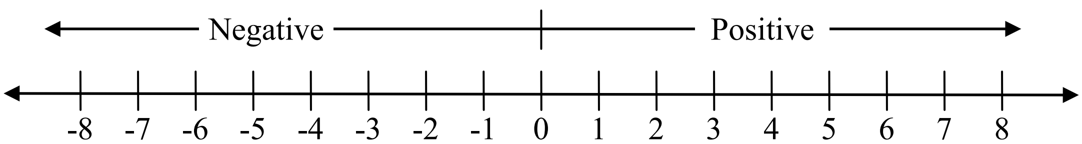
5.1 Signed Numbers

All the numbers we've looked at up until now have been **positive** numbers: numbers bigger than zero. If a number is less than zero, it is a **negative number**.

Example: The temperature is 20 degrees below 0°.
If the temperature was 30 degrees *above* 0°, we'd just write 30°.
Since the temperature is 20 degrees *below* 0°, we write -20°.

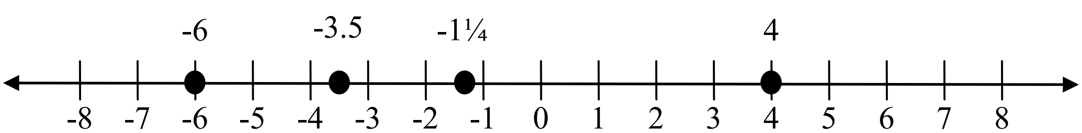
Example: Ben overdrew his bank account, and now owes them \$50.
Since his account balance is below \$0, we could write the balance as -\$50.

We can visualize negative numbers using a number line. Values increase as you move to the right and decrease to the left.



Every number has an **opposite**: a number on the other side of zero, the same distance from zero.
Example: Find the opposite of: a) 5 b) -3 c) ½
Since 5 is five units to the right of zero, the opposite is five units to the left: -5
Since -3 is three units to the left of zero, the opposite is three units to the right: 3
Since ½ to the right of zero, the opposite is to the left: -½

Example: Place these numbers on the number line: a) 4 b) -6 c) -3.5 d) -1¼
a) 4 is four units to the right of zero. b) -6 is six units to the left of zero
c) -3.5 is halfway between -3 and -4 d) -1¼ is further left than -1; it is the opposite of 1¼



We can compare two signed numbers by thinking about their location on the number line. A number further left on the number line is smaller than a number to its right.

Example: Write < or > to compare the numbers: a) 3 __ 5 b) -4 __ 3 c) -2 __ -5 d) -2.1 __ -2.4
a) On a number line, 3 is to the left of 5, so 3 < 5
b) On the number line, -4 is to the left of 3, so -4 < 3
c) On a number line, -2 is to the right of -5, so -2 > -5
d) On a number line, -2.1 is to the right of -2.4, so -2.1 > -2.4

Worksheet – 5.1 Signed Numbers Name: ________________________________

Write a signed number for each situation

1) I deposit $200 in my bank account

2) I withdraw $100 from my account

3) 20 feet above sea level

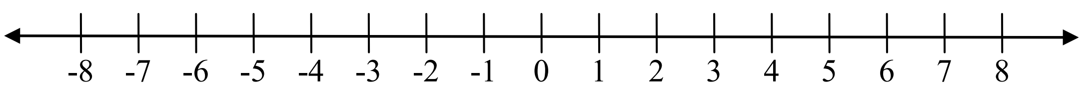
4) 40 feet below sea level

Find the opposite of each number

5) 3 6) -7 7) 3.7 8) -2.6 9) 3½ 10) -4¾

Place each number on the number line

11) -7 12) 4 13) -4.7 14) 3.2 15) 6¾ 16) -4¾



Write < or > to compare the numbers

17) 2 __ 8 18) 207 __ 198 19) 23 __ -37 20) -15 __ 34

21) -2 __ -7 22) -8 __ -4 23) -152 __ -130 24) -1743 __ -823

25) 2.3 __ 3.1 26) -5.3 __ -5.8 27) -3.8 __ 2.3 28) -0.3 __ -0.07

29) $\frac{2}{3}$ __ $\frac{1}{3}$ 30) $-\frac{2}{5}$ __ $-\frac{4}{5}$ 31) $-3\frac{1}{4}$ __ $-3\frac{3}{4}$ 32) -6 __ $-6\frac{1}{2}$

33) $\frac{2}{5}$ __ $\frac{1}{3}$ 34) $-\frac{5}{8}$ __ $-\frac{7}{10}$ 35) $1\frac{3}{4}$ __ $-1\frac{5}{7}$ 36) -1.2 __ $-1\frac{5}{12}$

5.2 Adding / Subtracting Signed Numbers

To add signed numbers of the **same sign** (both positive or both negative)

- Add the absolute values of the numbers
- If both numbers are negative, the sum is negative

Example: Add: $-8 + (-5)$
Since both numbers are negative, we add their absolute values: $8 + 5 = 13$
The result will be negative: $-8 + (-5) = -13$

To add signed numbers of **opposite sign** (one positive, one negative)

- Find the absolute value of each number
- Subtract the smaller absolute value from the larger value
- If the negative number had larger absolute value, the result will be negative

Example: Add: $-4 + 9$
The absolute values of the two numbers are 9 and 4. We subtract the smaller from the larger: $9 - 4 = 5$
Since 9 had the larger absolute value and is positive, the result will be positive. $9 + (-4) = 5$

Example: Add: $5 + (-8)$
The absolute values of the two numbers are 5 and 8. We subtract the smaller from the larger: $8 - 5 = 3$
Since 8 had the larger absolute value and is negative, the result will be negative. $5 + (-8) = -3$

Notice that $5 - 3$ is the same as $5 + (-3)$. Likewise, $5 - (-3)$ is the same as $5 + 3$. Subtracting a number is the same as adding its opposite. To **subtract** signed numbers:

- Rewrite subtraction as adding the opposite of the second number:
  $a - b = a + (-b)$ and $a - (-b) = a + b$

Example: Subtract: $10 - (-3)$
We rewrite the subtraction as adding the opposite: $10 + 3 = 13$

Example: Subtract: $-5.3 - 6.1$
We rewrite the subtraction as adding the opposite: $-5.3 + (-6.1)$
Since these have the same sign, we add their absolute values: $5.3 + 6.1 = 11.4$
Since both are negative the result is negative: $-5.3 - 6.1 = -11.4$

Example: Subtract: $\frac{1}{6} - \frac{2}{3}$

First, we give these a common denominator: $\frac{1}{6} - \frac{4}{6}$. Next, rewrite as adding the opposite: $\frac{1}{6} + \left(-\frac{4}{6}\right)$

Since these are opposite signs, we subtract the absolute values: $\frac{4}{6} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

Since the negative number had larger absolute value, the result is negative: $\frac{1}{6} - \frac{2}{3} = -\frac{1}{2}$

Worksheet – 5.2 Add / Subtract Signed Numbers Name: ________________________________

Add or Subtract:

1) $-8 + 3$

2) $-1 + 13$

3) $8 + (-6)$

4) $120 + (-150)$

5) $-14 + (-10)$

6) $7.1 + 3.6$

7) $-1.6 + 3.4$

8) $-0.4 + (-0.8)$

9) $18 - 6$

10) $6 - 18$

11) $23 - 67$

12) $-10 - 8$

13) $-10 - (-4)$

14) $26 - (-12)$

15) $8.3 - 10.4$

16) $-3.22 - 4$

17) $\frac{4}{5} - \frac{8}{5}$

18) $\frac{1}{12} - \frac{5}{6}$

19) $-2\frac{1}{2} - 4$

20) $5\frac{1}{4} - 3\frac{5}{6}$

21) In Fargo it was -18°F, while in Tacoma it was 43°F. How much warmer was Tacoma?

22) Darrel’s account was overdrawn by $120, before he deposited $450. What is his balance now?

5.3 Multiplying / Dividing Signed Numbers

To multiply or divide two signed numbers

- If the two numbers have **different sign**, the result will be negative
- If the two numbers have the **same sign**, the result will be positive

Example: Multiply: a) $-4 \cdot 3$ b) $5(-6)$ c) $-7(-4)$

a) The factors have different signs, so the result will be negative: $-4 \cdot 3 = -12$

b) The factors have different signs, so the result will be negative: $5(-6) = -30$

c) The factors have the same signs, so the result will be positive: $-7(-4) = 28$

Example: Divide: a) $-40 \div 10$ b) $8 \div (-4)$ c) $\dfrac{-36}{-3}$

a) The numbers have different signs, so the result will be negative: $-40 \div 10 = -4$

b) The numbers have different signs, so the result will be negative: $8 \div (-4) = -2$

c) The numbers have the same signs, so the result will be positive: $\dfrac{-36}{-3} = 12$

The same rules apply to fractions and decimals

Example: Calculate: a) $-8 \div 12$ b) $\dfrac{-18}{-4}$ c) $-\dfrac{2}{3} \cdot \dfrac{6}{7}$

a) Since this doesn't divide evenly, we can either use decimals or fractions. Using fractions,

$-8 \div 12 = \dfrac{-8}{12} = -\dfrac{2}{3}$.

b) Since both numbers have the same sign, the result will be positive: $\dfrac{-18}{-4} = \dfrac{9}{2} = 4\dfrac{1}{2}$

c) Different signs, so the result will be negative: $-\dfrac{2}{3} \cdot \dfrac{6}{7} = -\dfrac{2}{1} \cdot \dfrac{2}{7} = -\dfrac{4}{7}$

The same order of operations we used before also applies to signed numbers

Example: Simplify: $-3 - 5\left(3^2 + 6 \div (-2)\right)$

We begin with the inside of the parens, with the exponent:	$-3 - 5\left(9 + 32 \div (-2)\right)$
Still inside the parens, we do the division:	$-3 - 5\left(9 + (-16)\right)$
Inside the parens, we add	$-3 - 5(-7)$
Now multiply	$-3 - (-35)$
Rewrite as addition	$-3 + 35$
Add	32

Worksheet – 5.3 Multiply / Divide Signed Numbers   Name: ________________________________

Multiply or divide

1) $-7 \cdot 4$

2) $-5(-8)$

3) $5(-3)$

4) $-3.2(-6)$

5) $32 \div (-4)$

6) $-48 \div (-8)$

7) $-30 \div 3$

8) $-6.3 \div (-2.1)$

9) $\frac{-16}{-4}$

10) $\frac{-5}{10}$

11) $\frac{12}{-1}$

12) $\frac{-6.23}{-0.1}$

13) $\frac{3}{8} \cdot \left(-\frac{5}{6}\right)$

14) $-\frac{4}{5} \div \left(-\frac{3}{10}\right)$

15) $-10 \cdot \frac{3}{4}$

16) $-10 \cdot (-3)(-5)$

17) $\frac{4}{5} - \frac{8}{5}$

18) $\frac{1}{12} - \frac{5}{6}$

19) $-2\frac{1}{2} - 4$

20) $5\frac{1}{4} - 3\frac{5}{6}$

Simplify

21) $4 - (-5) \cdot 6$

22) $(-3)^2 - 4^2$

23) $\frac{3 - 4(5 - 7)}{1 + 6 \div 3}$

24) $\frac{1}{4} - \frac{3}{4} \cdot \frac{1}{6}$

5.5 Evaluating Formulas

Often we need to use formulas to solve problems. For example, you might remember the formula we had for perimeter of a rectangle: $P = 2L + 2W$.

Formulas contain ***variables*** - letters used to represent unknown quantities. In the formula above, P, L, and W are the variables. When we ***evaluate*** a formula, we substitute values for unknown quantities.

Example: Use the perimeter formula $P = 2L + 2W$ to find the perimeter of a rectangle 3 meters wide and 5 meters tall.
Since rectangle is 3 meters wide, we'll let W=3. Since it is 5 meters tall, we'll let L=5
Substituting those values in the formula, we get $P = 2(5)+2(3) = 10 + 6 = 16$. The perimeter is 16.

Example: Evaluate the formula $A = 3n - 2p$ when $n = 7$ and $p = -6$
Making those substitutions, $A = 3(7) - 2(-6) = 21 - (-12) = 21 + 12 = 33$

Example: Given the formula $y = \frac{2}{3}x^2 + 5$, find y when $x = -3$

Replacing x with –3 in the formula, we get: (notice we square x, so we square all of –3)

$y = \frac{2}{3}(-3)^2 + 5$ Squaring the -3,

$y = \frac{2}{3} \cdot 9 + 5$ Multiply. You can rewrite as $y = \frac{2}{3} \cdot \frac{9}{1} + 5$ if you want

$y = 6 + 5$

$y = 11$

To make formulas easier to work with, we use some rules to make the expressions simpler. You'll learn a lot more about this next quarter, but let's look at a couple.

Combining like terms is what we do when we combine similar things.

Example: If we see the formula $A = 3p + 2n + 5p - n$, you might notice that $3p$ and $5p$ have the same kind of thing: p's. If we have 3 of them and add 5 more, now we'll have 8 p's: $3p + 5p = 8p$.
Likewise, $2n - n = 1n = n$. Altogether,
$A = 3p + 2n + 5p - n$ can be simplified to
$A = 8p + n$.

Example: Simplify $2x - 4y + 2 + 6x - 5 + y$
We can combine the terms with x in them: $2x + 6x = 8x$.
We can combine the terms with y in them: $-4y + y = -3y$
We can combine the numbers: $2 - 5 = -3$
Altogether, $2x - 4y + 2 + 6x - 5 + y$ simplifies to $6x - 3y - 3$

Worksheet – 5.5 Evaluating Formulas

Name: ________________________________

Evaluate the formulas for the given values

1) Evaluate $I = P \cdot r \cdot t$ for $P = 300$, $r = 0.05$, $t = 10$

2) Evaluate $P = 200 + 30t$ when $t = 8$

3) Evaluate $z = \dfrac{x - \mu}{\sigma}$ when $x = 80, \mu = 100, \sigma = 15$

4) Evaluate $a - (a + b)(3 - a)$ when $a = 4$, $b = -2$

Simplify the formula by combining like terms.

5) $A = 6a - 2a + 3b + 4a$

6) $y = 2x + 5 - 3x$

7) $y = 6 - 200p + 6 - 20p$

8) $R = 200 + 15x - 100 - 10x$

Notice that $5(2+3) = 5 \cdot 2 + 5 \cdot 3$. This idea is called **distributing**. In general, $a(b + c) = a \cdot b + a \cdot c$

Example: $2(x+3) = 2 \cdot x + 2 \cdot 3 = 2x + 6$

Example: $2(x-3) = 2 \cdot x - 2 \cdot 3 = 2x - 6$

Example: $-(3x-2) = -1(3x-2) = -1 \cdot 3x - (-1) \cdot 2 = -3x - (-2) = -3x + 2$

or, $-(3x-2) = -1(3x + (-2)) = -1 \cdot 3x + (-1)(-2) = -3x + 2$

Use distribution and combining like terms to simplify each expression

9) $4(5 - p)$

10) $-2(3p + 4)$

11) $-3(5 - 2p)$

12) $4 - (2n - 3)$

13) $2x - 3(x + 1)$

14) $5x + 4(x - 1)$

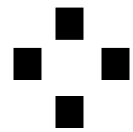
15) $2(3 - w) + 3w$

Pattern Building

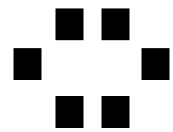
The goal of today is to work on seeing and representing patterns.
To do this, we're going to look at a progression of "steps" of pattern, and try to write down what we see, then find an expression that explains it.

Example: Consider the three steps to the right. How many blocks would be in Step 4? Step 10? Step n?

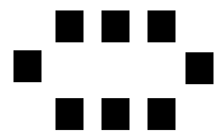
Step 1



Step 2



Step 3



We'll look at two different student's approaches.

Student 1 notices that all three steps shown have a single dot on the far left and far right, so that's 2 dots. There's a top row and bottom row of dots, each of which is increasing by 1 each time.
So in step 1, we have 2 dots + 2 rows of 1 dot each: $2 + 2\cdot 1$
We jot this down, and note the pattern, which we can then extend:

Step	What I See Here	Number of dots
1	$2 + 2 \cdot \mathbf{1}$	4
2	$2 + 2 \cdot \mathbf{2}$	6
3	$2 + 2 \cdot \mathbf{3}$	8
4	$2 + 2 \cdot \mathbf{4}$	10
10	$2 + 2 \cdot \mathbf{10}$	22
n	$2 + 2 \cdot \boldsymbol{n}$	$2 + 2n$

Student 2 notices that we start with 4 dots, and add 2 dots each time. So, in Step 2, we have 4 dots + 2 more. In step 3 we have 4 dots + 4 more, which is 2 more twice: $2\cdot 2$, or $4 + 2\cdot 2$
We jot this down, and note the pattern, which we can then extend:

Step	What I See Here	Number of dots
1	4, or $4 + 2 \cdot \mathbf{0}$	4
2	$4 + 2 \cdot \mathbf{1}$	6
3	$4 + 2 \cdot \mathbf{2}$	8
4	$4 + 2 \cdot \mathbf{3}$	10
10	$4 + 2 \cdot \mathbf{9}$	22
n	$4 + 2 \cdot (\boldsymbol{n} \mathbf{- 1})$	$4 + 2(n - 1)$

Is one of the students wrong? Or are their answers the same?
We can check by simplifying Student 2's answer:

$4 + 2(n - 1)$	Distributing
$4 + 2n - 2$	Combining like terms, $4 - 2 = 2$
$2 + 2n$	

The answers are the same, just written differently

Worksheet – Patterns Name: ________________________________

For each of the pattern sheets on the table, figure out What You See, and try to find out how many boxes will be needed for Step 4, Step 10, and Step *n*.

1)

Stage	What I See Here	Number of boxes
1		
2		
3		
4		
10		
n		

2)

Stage	What I See Here	Number of boxes
1		
2		
3		
4		
10		
n		

3)

Stage	What I See Here	Number of boxes
1		
2		
3		
4		
10		
n		