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## 2016 Mississippi <br> College- and CareerReadiness Standards for Mathematics

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# 2016 Mississippi College- and Career-Readiness Standards for Mathematics 

Carey M. Wright, Ed.D., State Superintendent of Education
Kim S. Benton, Ed.D., Chief Academic Officer Jean Massey, Executive Director, Office of Secondary Education

Nathan Oakley, Ph.D., Executive Director, Office of Elementary Education and Reading Trecina Green, Executive Director, Office of Professional Development Marla Davis, Ph.D., NBCT, Bureau Director, Office of Secondary Education

## Mississippi Department of Education Post Office Box 771 Jackson, Mississippi 39205-0771

# Office of Elementary Education and Reading Office of Secondary Education 

## 601-359-2586 www.mde.k12.ms.us/ESE

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Mississippi Department of Education
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# Introduction 

Mission Statement

The Mississippi Department of Education is dedicated to student success including the improvement of student achievement in mathematics in order to produce citizens who are capable of making complex decisions, solving complex problems, and communicating fluently in a technological society. The 2016 Mississippi College- and Career-Readiness Standards for Mathematics ("The Standards") provide a consistent, clear understanding of what students are expected to know and be able to do by the end of each grade level and course. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that students need for success in college and careers and to compete in the global economy.

## Purpose

In an effort to closely align instruction for students who are progressing toward postsecondary study and the workforce, the 2016 Mississippi College- and Career-Readiness Standards for Mathematics include grade- and course-specific standards for K-12 mathematics.

The primary purpose of this document is to provide a basis for curriculum development for Grades K-12 mathematics teachers, outlining what students should know and be able to do by the end of each grade level and course. Courses for grades K-12 are based on the Mississippi College- and Career-Readiness Standards (MS CCRS) for Mathematics. Mississippi-specific courses that were revised to align with the (MS CCRS) for Mathematics include Pre-Calculus (renamed Algebra III) and Calculus.

The Southern Regional Education Board (SREB) Math Ready course is included as a transition to college mathematics courses.

The content of this document is centered on the mathematics domains of Counting and Cardinality (Grade K), Operations and Algebraic Thinking; Numbers and Operations in Base Ten (Grades K-5); Numbers and Operations-Fractions (Grades 3-5); Measurement and Data (Grades K-5); Ratios and Proportional Relationships (Grades 6-7); the Number System, Expressions \& Equations, Geometry, Statistics \& Probability (Grades 6-8); Functions (Grade 8), and the high school conceptual categories of Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics \& Probability. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

The required year for the 2016 Mississippi College- and Career-Readiness Standards for Mathematics is school year 2016-2017.

## Technology

The Mississippi Department of Education (MDE) strongly encourages the use of technology in all mathematics classrooms. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

The appropriate use of instructional technology is integrated throughout the 2016 Mississippi College- and Career-Readiness Standards for Mathematics. Teaching strategies at each grade level and in every secondary course incorporate technology in the form of calculators, software, or on-line internet resources. The graphing calculator is an integral part of mathematics courses beginning with Grade 6.

The MDE believes strongly in the Principles and Standards for School Mathematics Technology Principle of the National Council for Teachers of Mathematics (NCTM):
"Calculators and computers are reshaping the mathematical landscape, and school mathematics should reflect those changes. Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalization or abstraction. In the mathematics classrooms envisioned in Principles and Standards, every student has access to technology to facilitate his or her mathematics learning. Technology also offers options for students with special needs. Some students may benefit from the more constrained and engaging task situations possible with computers. Students with physical challenges can become much more engaged in mathematics using special technologies. Technology cannot replace the mathematics teacher, nor can it be used as a replacement for basic understandings and intuitions. The teacher must make prudent decisions about when and how to use technology and should ensure that the technology is enhancing students' mathematical thinking."
(NCTM, 2013, http://www.nctm.org.)

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## College- and CareerReadiness Standards for Mathematics Overview

## College- and Career-Readiness Standards for Mathematics

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is "a mile wide and an inch deep." These Standards are a substantial answer to that challenge. Aiming for clarity and specificity, these Standards endeavor to follow a design that not only stresses conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the laws of arithmetic to structure those ideas.

## Understanding Mathematics

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a+b)(x+y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a+b+c)(x+y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific expectations but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary for college and/or careers. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with reading disabilities the use of Braille, screen reader technology, or other assistive devices should be made available. In addition, while writing, these students should have access to a scribe, computer, or speech-to-text technology in their classroom. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of College- and Career-Readiness for all students.

## Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## 2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize-to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents-and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argumentexplain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## 4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a
website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 6. Attend to precision

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well-remembered
$7 \times 5+7 \times 3$, in preparation for learning about the distributive property. In the expression $x^{2}+$ $9 x+14$, older students can see the 14 as $2 \times 7$ and the 9 as $2+7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5-3$ (x$y)^{2}$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.
8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1,2)$ with slope 3 , middle school students might abstract the equation $(y-2) /(x-1)=3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1),(x-1)\left(x^{2}+x+1\right)$, and $(x-1)\left(x^{3}+x^{2}+x+1\right)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the 2016 Mississippi Collegeand Career-Readiness Standards for Mathematics

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to the Standards in mathematics instruction.

The Standards are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

## Modeling (High School Courses only)

Modeling standards are noted throughout the high school courses with an asterisk (*). Modeling links classroom mathematics and statistics to everyday life, work, and decisionmaking. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.


Making mathematical models is a Standard for Mathematical Practice, and specific Modeling standards appear throughout the high school standards. The basic modeling cycle above involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.


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# College- and CareerReadiness Standards for Mathematics Scaffolding Document 

# 2016 Mississippi College- and Career-Readiness Standards Scaffolding Document 

Purpose

The primary purpose of the 2016 Mississippi College- and Career-Readiness Standards Scaffolding Document is to provide teachers with a deeper understanding of the Standards as they plan for classroom instruction. Based on the 2016 Mississippi College- and CareerReadiness Standards for Mathematics, this document provides a close analysis of the requirements for student mastery. Because of the rigor and depth of the Standards, scaffolding instruction to meet the needs of all learners is essential to individual success. The Scaffolding Document will aid teachers' understanding of how to teach the Standards through a natural progression of student mastery. The Scaffolding Document is located at http://www.mde.k12.ms.us/ESE/ccr.

Organization of the 2016 Mississippi College- and Career-Standards Scaffolding Document
The 2016 Mississippi College- and Career-Readiness Standards Scaffolding Document is divided by grade level. Within each grade level, the Scaffolding Document is separated into the mathematical domains Counting and Cardinality (Grade K), Operations and Algebraic Thinking (Grades K-5); Numbers and Operations in Base Ten (Grades K-5); Numbers and Operations-Fractions (Grades 3-5); Measurement and Data (Grades K-5); Ratios and Proportional Relationships (Grades 6-7); the Number System, Expressions \& Equations, Geometry, Statistics \& Probability (Grades 6-8); Functions (Grade 8), and the high school conceptual categories of Number and Quantity, Algebra, Functions, Geometry, and Statistics \& Probability.

Each standard is then broken down into three categories: Prerequisite Knowledge, Conceptual Understanding, and Evidence of Knowledge. The Prerequisite Knowledge column lists the skills that students should have mastered in previous grades in order to work towards mastery of the grade-specific standard. In other words, this column details what a student needs to KNOW before mastering the grade-specific standard. The Conceptual Understanding column explains the deeper understanding of concepts-not actions or skills-that are required for mastery of the grade specific standard. In other words, this column explains what a student needs to UNDERSTAND before mastering the grade-specific standard. The last column, Evidence of Knowledge, explains what student mastery looks like, including what work a student produces to exhibit mastery of the grade-specific standard. In other words, this column describes what a student needs to $D O$ to show mastery of the grade-specific standard.

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# College- and CareerReadiness Standards for Mathematics (Grades K-5) 

## Fluency/Fluently Defined

Throughout the 2016 Mississippi College- and Career-Readiness Standards for Mathematics Grades K-5 standards, the words fluency and fluently will appear in bold, italicized, and underlined font (for example: $\underline{f l u e n t l y}$ ). With respect to student performance and effective inclass instruction, the expectations for mathematical fluency are explained below:

Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend one or more grades earlier in the standards than the grade when fluency is finally expected.

Wherever the word fluently appears in a MS CCR content standard, the word means quickly and accurately. It is important to understand that this is not explicitly tied to assessment purposes, but means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow: Fluent isn't halting, stumbling, or reversing oneself.

A key aspect of fluency in this sense is that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all students to meet the standards that call explicitly for fluency.


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## College- and Career-Readiness Standards for Mathematics (Grades 6-8)

## Grade 6

In Grade 6, instruction should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking. Each critical area is described below.
(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.
(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.
(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple onestep equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3 x=y$ ) to describe relationships between quantities.
(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for

## Grade 6 (continued)

summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

The content of this document is centered on the mathematics domains of Counting and Cardinality (Grade K), Operations and Algebraic Thinking; Numbers and Operations in Base Ten (Grades K-5); Numbers and Operations-Fractions (Grades 3-5); Measurement and Data (Grades K-5); Ratios and Proportional Relationships (Grades 67); the Number System, Expressions \& Equations, Geometry, Statistics \& Probability (Grades 6-8); Functions (Grade 8), and the high school conceptual categories of Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics \&
Probability. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

## Grade 6

## Ratios and Proportional Relationships (RP)

## Understand ratio concepts and use ratio reasoning to solve problems

| $6 . R P .1$ | Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate $A$ received, candidate $C$ received nearly three votes." |
| :---: | :---: |
| 6.RP. 2 | Understand the concept of a unit rate $a / b$ associated with a ratio $a: b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3 / 4$ cup of flour for each cup of sugar." "We paid $\$ 75$ for 15 hamburgers, which is a rate of $\$ 5$ per hamburger."1 |
| 6.RP. 3 | Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. <br> a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. <br> b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? <br> c. Find a percent of a quantity as a rate per 100 (e.g., $30 \%$ of a quantity means $30 / 100$ times the quantity); solve problems involving finding the whole, given a part and the percent. <br> d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. |
|  | The Number System (NS) |

## Apply and extend previous understandings of multiplication and division to divide fractions by fractions

|  | Interpret and compute quotients of fractions, and solve word problems involving division of <br> fractions by fractions, e.g., by using visual fraction models and equations to represent the <br> problem. For example, create a story context for $(2 / 3) \div(3 / 4)$ and use a visual fraction model to <br> show the quotient; use the relationship between multiplication and division to explain that $(2 / 3)$ <br> 6.NS. <br> (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) $\div(c / d)=$ ad/bc.) How much chocolate will <br> each person get if 3 people share 1/2 Ib of chocolate equally? How many 3/4-cup servings are in <br> 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area $1 / 2$ <br> square mi? |
| :--- | :--- |
|  | Compute fluently with multi-digit numbers and find common factors and multiples |
| 6.NS.2 | Fluently divide multi-digit numbers using the standard algorithm. |
| 6.NS.3 | Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for <br> each operation. |

## Grade 6

| 6.NS. 4 | Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12 . Use the distributive property to express a sum of two whole numbers $1-100$ with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express $36+8$ as $4(9+2)$. |
| :---: | :---: |
| Apply and extend previous understandings of numbers to the system of rational numbers |  |
| 6.NS. 5 | Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. |
| 6.NS. 6 | Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. <br> a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3)=3$, and that 0 is its own opposite. <br> b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. <br> c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. |
| 6.NS. 7 | Understand ordering and absolute value of rational numbers. <br> a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3>-7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right. <br> b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write $-3^{\circ} \mathrm{C}>-7^{\circ} \mathrm{C}$ to express the fact that $-3^{\circ} \mathrm{C}$ is warmer than $-7^{\circ} \mathrm{C}$. <br> c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write $\|-30\|=30$ to describe the size of the debt in dollars. <br> d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than - 30 dollars represents a debt greater than 30 dollars. |
| 6.NS. 8 | Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. |

## Grade 6

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Apply and extend previous understandings of addition and subtraction to add and subtract integers; represent addition and subtraction on a horizontal or vertical number line diagram.
a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
b. Understand $p+q$ as the number located a distance $|q|$ from $p$, in the positive or negative direction depending on whether $q$ is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of integers by describing real-world contexts.
c. Understand subtraction of integers as adding the additive inverse, $p-q=p+(-q)$. Show that the distance between two integers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
d. Apply properties of operations as strategies to add and subtract integers.

## Expressions and Equations (EE)

## Apply and extend previous understandings of arithmetic to algebraic expressions

| $6 . E E .1$ | Write and evaluate numerical expressions involving whole-number exponents. |
| :--- | :--- |
| Write, read, and evaluate expressions in which letters stand for numbers. |  |
| a. Write expressions that record operations with numbers and with letters standing for |  |
| numbers. For example, express the calculation "Subtract y from 5" as 5-y. |  |
| b. Identify parts of an expression using mathematical terms (sum, term, product, factor, |  |
| quotient, coefficient); view one or more parts of an expression as a single entity. For |  |
| example, describe the expression $2(8+7)$ as a product of two factors; view (8 + 7) as |  |
| both a single entity and a sum of two terms. |  |
| c. Evaluate expressions at specific values of their variables. Include expressions that arise |  |
| from formulas used in real-world problems. Perform arithmetic operations, including those |  |
| involving whole-number exponents, in the conventional order when there are no |  |
| parentheses to specify a particular order (Order of Operations). For example, use the |  |
| formulas $V=s^{3}$ and $A=6 s^{2}$ to find the volume and surface area of a cube with sides of |  |
| length $s=1 / 2$. |  |

## Reason about and solve one-variable equations and inequalities

| 6.EE.5 | Understand solving an equation or inequality as a process of answering a question: which <br> values from a specified set, if any, make the equation or inequality true? Use substitution to <br> determine whether a given number in a specified set makes an equation or inequality true. |
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| 6.EE.6 | Use variables to represent numbers and write expressions when solving a real-world or <br> mathematical problem; understand that a variable can represent an unknown number, or, <br> depending on the purpose at hand, any number in a specified set. |

## Grade 6

| 6.EE. 7 | Solve real-world and mathematical problems by writing and solving equations of the form <br> $x+p=q$ and $p x=q$ for cases in which $p, q$ and $x$ are all nonnegative rational numbers. |
| :--- | :--- |
| 6.EE. 8 | Write an inequality of the form $x>c$ or $x<c$ to represent a constraint or condition in a real-world or <br> mathematical problem. Recognize that inequalities of the form $x>c$ or $x<c$ have infinitely many <br> solutions; represent solutions of such inequalities on number line diagrams. |

Represent and analyze quantitative relationships between dependent and independent variables

| 6.EE. 9 | Use variables to represent two quantities in a real-world problem that change in relationship to one <br> another. <br> - Write an equation to express one quantity, thought of as the dependent variable, in terms of <br> the other quantity, thought of as the independent variable. <br> - Analyze the relationship between the dependent and independent variables using graphs <br> and tables, and relate these to the equation. |
| :---: | :---: | :---: |
| For example, in a problem involving motion at constant speed, list and graph ordered pairs of <br> distances and times, and write the equation $d=65 t ~ t o ~ r e p r e s e n t ~ t h e ~ r e l a t i o n s h i p ~ b e t w e e n ~ d i s t a n c e ~$ <br> and time. |  |

## Geometry (G)

## Solve real-world and mathematical problems involving area, surface area, and volume

| 6.G.1 | Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing <br> into rectangles or decomposing into triangles and other shapes; apply these techniques in the <br> context of solving real-world and mathematical problems. |
| :--- | :--- |
| $6 . G .2$ | Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit <br> cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would <br> be found by multiplying the edge lengths of the prism. Apply the formulas $V=/ w h$ and <br> $V=$ bh to find volumes of right rectangular prisms with fractional edge lengths in the context of <br> solving real-world and mathematical problems. |
| $6 . G .3$ | Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find <br> the length of a side joining points with the same first coordinate or the same second coordinate. <br> Apply these techniques in the context of solving real-world and mathematical problems. |
| 6.G.4 | Represent three-dimensional figures using nets made up of rectangles and triangles, and use the <br> nets to find the surface area of these figures. Apply these techniques in the context of solving real- <br> world and mathematical problems. |
| $\quad$ Statistics and Probability (SP) |  |

## Grade 6

## Summarize and describe distributions

6.SP.4 $\quad$ Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Summarize numerical data sets in relation to their context, such as by:
a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
6.SP. 5
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

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## Additional Resource

## 2016 Mississippi College- and Career-Standards Scaffolding Document

The primary purpose of the 2016 Mississippi College- and Career-Readiness Standards Scaffolding Document is to provide teachers with a deeper understanding of the Standards as they plan for classroom instruction. Based on the 2016 Mississippi College- and CareerReadiness Standards for Mathematics, this document provides a close analysis of the requirements for student mastery. Because of the rigor and depth of the Standards, scaffolding instruction to meet the needs of all learners is essential to individual success. The Scaffolding Document will aid teachers' understanding of how to teach the Standards through a natural progression of student mastery. The Scaffolding Document can be found at http://www.mde.k12.ms.us/ESE/ccr.

## Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

[^0]:    ${ }^{1}$ Expectations for unit rates in this grade are limited to non-complex fractions.

