Bridge Math | B

Bridge Math is a course intended to build upon concepts taught in previous courses to allow students to gain a deeper knowledge of the real and complex number systems as well as the structure, use, and application of equations, expressions, and functions. Functions emphasized include linear, quadratic and polynomial. Students continue mastery of geometric concepts such as similarity, congruence, right triangles, and circles. Students use categorical and quantitative data to model real life situations and rules of probability to compute probabilities of compound events.

Bridge Math includes the following domains and clusters:

- **The Real Number System**
  - Use properties of rational and irrational numbers.
- **Quantities**
  - Reason quantitatively and use units to solve problems.
- **The Complex Number System**
  - Perform arithmetic operations with complex numbers.
- **Seeing Structure in Expressions**
  - Write expressions in equivalent forms to solve problems.
- **Arithmetic with Polynomials and Rational Expressions**
  - Perform arithmetic operations on polynomials.
  - Understand the relationship between zeros and factors of polynomials.
- **Creating Equations**
  - Create equations that describe numbers or relationships.
- **Reasoning with Equations and Inequalities**
  - Understand solving equations as a process of reasoning and explain the reasoning.
  - Solve equations and inequalities in one variable.
  - Solve systems of equations.
  - Represent and solve equations and inequalities graphically.
- **Interpreting Functions**
  - Understand the concept of a function and use function notation.
  - Interpret functions that arise in applications in terms of the context.
  - Analyze functions using different representations
- **Similarity, Right Triangles, and Trigonometry**
  - Understand similarity in terms of similarity transformations.
  - Define trigonometric ratios and solve problems involving right triangles.
- **Circles**
  - Find arc lengths and areas of sectors of circles.
- **Geometric Measurement and Dimension**
  - Visualize relationships between two-dimensional and three-dimensional objects.
- **Modeling with Geometry**
  - Apply geometric concepts in modeling situations.
- **Interpreting Categorical and Quantitative Data**
  - Summarize, represent, and interpret data on a single count or measurement variable.
  - Summarize, represent, and interpret data on two categorical and quantitative variables.
  - Interpret linear models.
- **Conditional probability and the Rules of Probability**
  - Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Mathematical Modeling

Mathematical Modeling is a Standard for Mathematical Practice (MP4) and a Conceptual Category. Specific modeling standards appear throughout the high school standards indicated with a star (★). Where an entire domain is marked with a star, each standard in that domain is a modeling standard.

Standards for Mathematical Practice

Being successful in mathematics requires the development of approaches, practices, and habits of mind that need to be in place as one strives to develop mathematical fluency, procedural skills, and conceptual understanding. The Standards for Mathematical Practice are meant to address these areas of expertise that teachers should seek to develop in their students. These approaches, practices, and habits of mind can be summarized as "processes and proficiencies" that successful mathematicians have as a part of their work in mathematics. Additional explanations are included in the main introduction of these standards.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Literacy Standards for Mathematics

Communication in mathematics employs literacy skills in reading, vocabulary, speaking and listening, and writing. Mathematically proficient students communicate using precise terminology and multiple representations including graphs, tables, charts, and diagrams. By describing and contextualizing mathematics, students create arguments and support conclusions. They evaluate and critique the reasoning of others, analyze, and reflect on their own thought processes. Mathematically proficient students have the capacity to engage fully with mathematics in context by posing questions, choosing appropriate problem-solving approaches, and justifying solutions. Further explanations are included in the main introduction.

Literacy Skills for Mathematical Proficiency
1. Use multiple reading strategies. 2. Understand and use correct mathematical vocabulary. 3. Discuss and articulate mathematical ideas. 4. Write mathematical arguments.

Number and Quantity

The Real Number System (N.RN)

Cluster Headings	Content Standards
A. Use properties of rational and irrational numbers.	**B.N.RN.A.1**. Use rational and irrational numbers in calculations and in real-world context.

Quantities★ (N.Q)

Cluster Headings	Content Standards
A. Reason quantitatively and use units to solve problems.	**B.N.Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **B.N.Q.A.2** Define appropriate quantities for the purpose of descriptive modeling. **B.N.Q.A.3** Solve problems involving squares, square roots of numbers, cubes, and cube roots of numbers.

The Complex Number System (N.CN)

Cluster Headings	Content Standards
A. Perform arithmetic operations with complex numbers.	**B.N.CN.A.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. **B.N.CN.A.2** Know and use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Algebra

Seeing Structure in Expressions (A.SSE)

Cluster Headings	Content Standards
A. Write expressions in equivalent forms to solve problems.	**B.A.SSE.A.1** Use properties of multiplication and division to solve problems containing scientific notation. **B.A.SSE.A.2** Use algebraic structures to solve problems involving proportional reasoning in real-world context.

Arithmetic with Polynomials and Rational Expressions (A.APR)

Cluster Headings	Content Standards
A. Perform arithmetic operations on polynomials.	**B.A.APR.A.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
B. Understand the relationship between zeros and factors of polynomials.	**B.A.APR.B.2** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Creating Equations★ (A.CED)

Cluster Headings	Content Standards
A. Create equations that describe numbers or relationships.	**B.A.CED.A.1** Create equations and inequalities in one variable and use them to solve real-world problems. **B.A.CED.A.2** Create equations in two or more variables to represent relationships between quantities. **B.A.CED.A.3** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Reasoning with Equations and Inequalities (A.REI)

Cluster Headings	Content Standards
A. Understand solving equations as a process of reasoning and explain the reasoning.	**B.A.REI.A.1** Build functions and write expressions, equations, and inequalities for common algebra settings leading to a solution in context (e.g., rate and distance problems and problems that can be solved using proportions).
B. Solve equations and inequalities in one variable.	**B.A.REI.B.2** Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
C. Solve systems of equations.	**B.A.REI.C.3** Solve and explain the solutions to a system of equations using a variety of representations including combinations of linear and non-linear equations.
D. Represent and solve equations and inequalities graphically.	**B.A.REI.D.4** Use algebra and geometry to solve problems involving midpoints and distances. **B.A.REI.D.5** Solve a linear inequality using multiple methods and interpret the solution as it applies to the context.

Functions

Interpreting Functions (F.IF)

Cluster Headings	Content Standards
A. Understand the concept of a function and use function notation.	**B.F.IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. **B.F.IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Cluster Headings	Content Standards
B. Interpret functions that arise in applications in terms of the context.	**B.F.IF.B.3** Recognize functions as mappings of an independent variable into a dependent variable. ⋆
C. Analyze functions using different representations.	**B.F.IF.C.4** Graph linear, quadratic, absolute value, and piecewise functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated ones. ⋆ **B.F.IF.C.5** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **B.F.IF.C.6** Use the properties of exponents to interpret expressions for exponential functions.

Geometry

Similarity, Right Triangles and Trigonometry (G.SRT)

Cluster Headings	Content Standards
A. Understand similarity in terms of similarity transformations.	**B.G.SRT.A.1** Apply similar triangles to solve problems, such as finding heights and distances.
B. Define trigonometric ratios and solve problems involving right triangles.	**B.G.SRT.B.2** Apply basic trigonometric ratios to solve right triangle problems. **B.G.SRT.B.3** Apply properties of 30° 60° 90°, 45° 45° 90°, similar, and congruent triangles. **B.G.SRT.B.4** Solve problems involving angles of elevation and angles of depression.

Circles (G.C)

Cluster Headings	Content Standards
A. Find arc lengths and areas of sectors of circles.	**B.G.C.A.1** Apply a variety of strategies to determine the area and circumference of circles after identifying necessary information.

Geometric Measurement and Dimension (G.GMD)

Cluster Headings	Content Standards
A. Visualize relationships between two-dimensional and three-dimensional objects.	**B.G.GMD.A.1** Use relationships involving area, perimeter, and volume of geometric figures to compute another measure. **B.G.GMD.A.2** Use several angle properties to find an unknown angle measure. **B.G.GMD.A.3** Apply a variety of strategies using relationships between perimeter, area, and volume to calculate desired measures in composite figures (i.e., combinations of basic figures).

Modeling with Geometry (G.MG)

Cluster Headings	Content Standards
A. Apply geometric concepts in modeling situations.	**B.G.MG.A.1** Use appropriate technology to find the mathematical model for a set of non-linear data. **B.G.MG.A.2** Solve problems involving surface area and volume in real-world context.

Statistics and Probability

Interpreting Categorical and Quantitative Data (S.ID)

Cluster Headings	Content Standards
A. Summarize, represent, and interpret data on a single count or measurement variable.	**B.S.ID.A.1** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
B. Summarize, represent, and interpret data on two categorical and quantitative variables.	**B.S.ID.B.2** Interpret and use data from tables, charts, and graphs.

Cluster Headings	Content Standards
B. Summarize, represent, and interpret data on two categorical and quantitative variables.	**B.S.ID.B.3** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **a**. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
C. Interpret linear models.	**B.S.ID.C.4** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Conditional Probability and the Rules of Probability (S.CP)

Cluster Headings	Content Standards
A. Use the rules of probability to compute probabilities of compound events in a uniform probability model.	**B.S.CP.A.1** Understand and use basic counting techniques in contextual settings. **B.S.CP.A.2** Compute a probability when the event and/or sample space are not given or obvious. **B.S.CP.A.3** Recognize the concepts of conditional and joint probability expressed in real-world contexts. **B.S.CP.A.4** Recognize the concept of independence expressed in real-world contexts.

Precalculus | P

Precalculus is designed to prepare students for college level STEM focused courses. Students extend their knowledge of the complex number system to use complex numbers in polynomial identities and equations. Topics for student mastery include vectors and matrix quantities, sequences and series, parametric equations, and conic sections. Students use previous knowledge to continue progressing in their understanding of trigonometric functions and using regression equations to model quantitative data.

Precalculus includes the following domains and clusters:

- **Number Expressions**
  - Represent, interpret, compare, and simplify number expressions.
- **The Complex Number System**
  - Perform complex number arithmetic and understand the representation on the complex plane.
  - Use complex numbers in polynomial identities and equations.
- **Vectors and Matrix Quantities**
  - Represent and model with vector quantities.
  - Understand the graphic representation of vectors and vector arithmetic.
  - Perform operations on matrices and use matrices in applications.
- **Sequences and Series**
  - Understand and use sequences and series.
- **Reasoning with Equations and Inequalities**
  - Solve systems of equations and nonlinear inequalities.
- **Parametric Equations**
  - Describe and use parametric equations.
- **Conic Sections**
  - Understand the properties of conic sections and apply them to model real-world phenomena.
- **Building Functions**
  - Build new functions from existing functions.
- **Interpreting Functions**
  - Analyze functions using different representations.
- **Trigonometric Functions**
  - Extend the domain of trigonometric functions using the unit circle.
- **Graphing Trigonometric Functions**
  - Model periodic phenomena with trigonometric functions.
- **Applied Trigonometry**
  - Use trigonometry to solve problems.
- **Trigonometric Identities**
  - Apply trigonometric identities to rewrite expressions and solve equations.
- **Polar Coordinates**
  - Use polar coordinates.
- **Model with Data**
  - Model data using regression equations.

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Number and Quantity

Number Expressions (N.NE)

Cluster Headings	Content Standards
A. Represent, interpret, compare, and simplify number expressions.	**P.N.NE.A.1** Use the laws of exponents and logarithms to expand or collect terms in expressions; simplify expressions or modify them in order to analyze them or compare them. **P.N.NE.A.2** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.★ **P.N.NE.A.3** Classify real numbers and order real numbers that include transcendental expressions, including roots and fractions of π and e. **P.N.NE.A.4** Simplify complex radical and rational expressions; discuss and display understanding that rational numbers are dense in the real numbers and the integers are not. **P.N.NE.A.5** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

The Complex Number System (N.CN)

Cluster Headings	Content Standards
A. Perform complex number arithmetic and understand the representation on the complex plane.	**P.N.CN.A.1** Perform arithmetic operations with complex numbers expressing answers in the form $a + bi$. **P.N.CN.A.2** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **P.N.CN.A.3** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **P.N.CN.A.4** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + 3i)^3 = 8$ *because* $(-1 + 3i)$ *has modulus 2 and argument 120°.*

A. Perform complex number arithmetic and understand the representation on the complex plane.	**P.N.CN.A.5** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
B. Use complex numbers in polynomial identities and equations.	**P.N.CN.B.6** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$. **P.N.CN.B.7** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities (N.VM)

Cluster Headings	Content Standards
A. Represent and model with vector quantities.	**P.N.VM.A.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes *(e.g.,* $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lVert\boldsymbol{v}\rVert$, v*)*. **P.N.VM.A.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **P.N.VM.A.3** Solve problems involving velocity and other quantities that can be represented by vectors.
B. Understand the graphic representation of vectors and vector arithmetic.	**P.N.VM.B.4** Add and subtract vectors. **a.** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **b.** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **c.** Understand vector subtraction $\boldsymbol{v} – \boldsymbol{w}$ as $\boldsymbol{v} + (–\boldsymbol{w})$, where $–\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
	P.N.VM.B.5 Multiply a vector by a scalar. **a.** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. **b.** Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\lVert c\boldsymbol{v}\rVert = \lvert c\rvert v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\lvert c\rvert v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Cluster Headings	Content Standards
B. Understand the graphic representation of vectors and vector arithmetic.	**P.N.VM.B.6** Calculate and interpret the dot product of two vectors.
C. Perform operations on matrices and use matrices in applications.	**P.N.VM.C.7** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. **P.N.VM.C.8** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **P.N.VM.C.9** Add, subtract, and multiply matrices of appropriate dimensions. **P.N.VM.C.10** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **P.N.VM.C.11** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **P.N.VM.C.12** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **P.N.VM.C.13** Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Algebra

Sequences and Series (A.S)

A. Understand and use sequences and series.	**P.A.S.A.1** Demonstrate an understanding of sequences by representing them recursively and explicitly.
	P.A.S.A.2 Use sigma notation to represent a series; expand and collect expressions in both finite and infinite settings.
	P.A.S.A.3 Derive and use the formulas for the general term and summation of finite or infinite arithmetic and geometric series, if they exist. a. Determine whether a given arithmetic or geometric series converges or diverges. b. Find the sum of a given geometric series (both infinite and finite). c. Find the sum of a finite arithmetic series.

A. Understand and use sequences and series.	**P.A.S.A.4** Understand that series represent the approximation of a number when truncated; estimate truncation error in specific examples.
	P.A.S.A.5 Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Reasoning with Equations and Inequalities (A.REI)

A. Solve systems of equations and nonlinear inequalities.	**P.A.REI.A.1** Represent a system of linear equations as a single matrix equation in a vector variable. **P.A.REI.A.2** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). **P.A.REI.A.3** Solve nonlinear inequalities (quadratic, trigonometric, conic, exponential, logarithmic, and rational) by graphing (solutions in interval notation if one-variable), by hand and with appropriate technology. **P.A.REI.A.4** Solve systems of nonlinear inequalities by graphing.

Parametric Equations (A.PE)

A. Describe and use parametric equations. ★	**P.A.PE.A.1** Graph curves parametrically (by hand and with appropriate technology). **P.A.PE.A.2** Eliminate parameters by rewriting parametric equations as a single equation.

Conic Sections (A.C)

A. Understand the properties of conic sections and model real-world phenomena.	**P.A.C.A.1** Display all of the conic sections as portions of a cone. **P.A.C.A.2** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **P.A.C.A.3** From an equation in standard form, graph the appropriate conic section: ellipses, hyperbolas, circles, and parabolas. Demonstrate an understanding of the relationship between their standard algebraic form and the graphical characteristics. **P.A.C.A.4** Transform equations of conic sections to convert between general and standard form.

Functions

Building Functions (F.BF)

Cluster Headings	Content Standards
A. Build new functions from existing functions.	**P.F.BF.A.1** Understand how the algebraic properties of an equation transform the geometric properties of its graph. *For example, given a function, describe the transformation of the graph resulting from the manipulation of the algebraic properties of the equation (i.e., translations, stretches, reflections, and changes in periodicity and amplitude).*
	P.F.BF.A.2 Develop an understanding of functions as elements that can be operated upon to get new functions: addition, subtraction, multiplication, division, and composition of functions.
	P.F.BF.A.3 Compose functions. *For example, if* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather balloon as a function of time, then* T(h(t)) *is the temperature at the location of the weather balloon as a function of time.*
	P.F.BF.A.4 Construct the difference quotient for a given function and simplify the resulting expression.
	P.F.BF.A.5 Find inverse functions (including exponential, logarithmic, and trigonometric). **a.** Calculate the inverse of a function, $f(x)$, with respect to each of the functional operations; in other words, the additive inverse, $-f(x)$, the multiplicative inverse, $1 / f(x)$, and the inverse with respect to composition, $f^{-1}(x)$. Understand the algebraic and graphical implications of each type. **b.** Verify by composition that one function is the inverse of another. **c.** Read values of an inverse function from a graph or a table, given that the function has an inverse. **d.** Recognize a function is invertible if and only if it is one-to-one. Produce an invertible function from a non-invertible function by restricting the domain.
	P.F.BF.A.6 Explain why the graph of a function and its inverse are reflections of one another over the line $y = x$.

Interpreting Functions (F.IF)

Cluster Headings	Content Standards
A. Analyze functions using different representations.	**P.F.IF.A.1** Determine whether a function is even, odd, or neither. **P.F.IF.A.2** Analyze qualities of exponential, polynomial, logarithmic, trigonometric, and rational functions and solve real-world problems that can be modeled with these functions (by hand and with appropriate technology).★ **P.F.IF.A.4** Identify the real zeros of a function and explain the relationship between the real zeros and the x-intercepts of the graph of a function (exponential, polynomial, logarithmic, trigonometric, and rational). **P.F.IF.A.5** Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$. **P.F.IF.A.6** Visually locate critical points on the graphs of functions and determine if each critical point is a minimum, a maximum, or point of inflection. Describe intervals where the function is increasing or decreasing and where different types of concavity occur. **P.F.IF.A.7** Graph rational functions, identifying zeros, asymptotes (including slant), and holes (when suitable factorizations are available) and showing end-behavior. **P.F.IF.A.8** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n - 1) for n ≥ 1.*

Trigonometric Functions (F.TF)

Cluster Headings	Content Standards
A. Extend the domain of trigonometric functions using the unit circle.	**P.F.TF.A.1** Convert from radians to degrees and from degrees to radians. **P.F.TF.A.2** Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–*x*, π+*x*, and 2π–*x* in terms of their values for *x*, where *x* is any real number. **P.F.TF.A.3** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **P.F.TF.A.4** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Graphing Trigonometric Functions (F.GT)

Cluster Headings	Content Standards
A. Model periodic phenomena with trigonometric functions.★	**P.F.GT.A.1** Interpret transformations of trigonometric functions. **P.F.GT.A.2** Determine the difference made by choice of units for angle measurement when graphing a trigonometric function. **P.F.GT.A.3** Graph the six trigonometric functions and identify characteristics such as period, amplitude, phase shift, and asymptotes. **P.F.GT.A.4** Find values of inverse trigonometric expressions (including compositions), applying appropriate domain and range restrictions. **P.F.GT.A.5** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **P.F.GT.A.6** Determine the appropriate domain and corresponding range for each of the inverse trigonometric functions. **P.F.GT.A.7** Graph the inverse trigonometric functions and identify their key characteristics. **P.F.GT.A.8** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Geometry

Applied Trigonometry (G.AT)

Cluster Headings	Content Standards
A. Use trigonometry to solve problems. ★	**P.G.AT.A.1** Use the definitions of the six trigonometric ratios as ratios of sides in a right triangle to solve problems about lengths of sides and measures of angles. **P.G.AT.A.2** Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **P.G.AT.A.3** Derive and apply the formulas for the area of sector of a circle. **P.G.AT.A.4** Calculate the arc length of a circle subtended by a central angle.

Cluster Headings	Content Standards
A. Use trigonometry to solve problems. ★	**P.G.AT.A.5** Prove the Laws of Sines and Cosines and use them to solve problems. **P.G.AT.A.6** Understand and apply the Law of Sines (including the ambiguous case) and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Trigonometric Identities (G.TI)

Cluster Headings	Content Standards
A. Apply trigonometric identities to rewrite expressions and solve equations.★	**P.G.TI.A.1** Apply trigonometric identities to verify identities and solve equations. Identities include: Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle. **P.G.TI.A.2** Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Polar Coordinates (G.PC)

Cluster Headings	Content Standards
A. Use polar coordinates.	**P.G.PC.A.1** Graph functions in polar coordinates. **P.G.PC.A.2** Convert between rectangular and polar coordinates. **P.G.PC.A.3** Represent situations and solve problems involving polar coordinates.★

Statistics and Probability

Model with Data★ (S.MD)

Cluster Headings	Content Standards
A. Model data using regressions equations.	**P.S.MD.A.1** Create scatter plots, analyze patterns, and describe relationships for bivariate data (linear, polynomial, trigonometric, or exponential) to model real-world phenomena and to make predictions. **P.S.MD.A.2** Determine a regression equation to model a set of bivariate data. Justify why this equation best fits the data.

A. Model data using regressions equations.	**P.S.MD.A.3** Use a regression equation, modeling bivariate data, to make predictions. Identify possible considerations regarding the accuracy of predictions when interpolating or extrapolating.